# UNIT 7: PROBABILITY

Use theoretical and experimental probability to model and solve problems.

a) Use addition and multiplication principles.
b) Calculate and apply permutations and combinations.
c) Create and use simulations for probability models.
d) Find expected values and determine fairness.

Identify and use discrete random variables to solve problems

| Day | Topic/Objective | Activity |
|---|---|---|
| 1 | • 13.1 Counting Principle (1.03 a) | |
| 2 | • 13.2 Permutations and Combinations (1.03 b) | |
| 3 | • 13.2 continued (1.03 b) | |
| 4 | • Review | **Quiz Days 1-3** |
| 5 | • Law of Large Numbers<br>• Empirical vs. Theoretical Probability (1.03 c)<br>• Basic Probability | |
| 6 | • 13.3 Probability<br>• Mutually Exclusive<br>• Independent/Dependent Events (1.03 b, c, e) | |
| 7 | • 13.4 Expected Value<br>• Fairness (1.03 d) | |
| 8 | • Geometric Probability | |
| 9 | • Binomial Probability | |
| 10 | • Review Day | |
| 11 | • TEST | **TEST** |

**Day 1: Fundamental Counting Principal**

Basic Counting Methods for Determining the Possible Outcomes

A. Fundamental Counting Principle:

a. Tree Diagrams:

Example #1: Alpo manufactures 4 different types of dog food: puppy, adult, active and senior. Each type comes in two different sizes: 8 lbs or 16 lbs. Make a tree diagram representing the different products. How many different products can the company display?

b. In general:

- If there are m ways to make a first selection and n ways to make a second selection, then there are ______________ ways to make the two selections simultaneously. This is called the Fundamental Counting Principle.

Example #1 above: 4 different types of dog food in 2 different sizes. How many different products?

Example #2: Joey has to pick out an outfit for school. He has 2 pairs of pants appropriate for school: one blue, one black. He has 3 shirts: one red, one green, one blue. He has 2 pairs of shoes to choose from: one blue, one black. How many different outfits can Joey select?

Example #3 (more restricted) Telephone numbers, in US, begin with three-digit area codes followed by seven-digit local telephone numbers. How many different telephone numbers are possible? (Area codes and local telephone numbers cannot begin with 0 or 1)

Example #4 In a certain state, automobile license plates display 3 letters followed by 3 digits. How many such plates are possible if repetition of the letters is

(a) allowed? (b) not allowed?

Example #5 In how many different ways can a race with six runners be completed?  Assume there is no tie.

Example #6 Three digit numbers are formed using the digits 2, 4, 5, and 7, with repetition of digits allowed.  How many such members can be formed if

(a) the numbers are less than 700?

(b) the numbers are even?

(c) the numbers are divisible by 5?

(d) the number must start with a 2?

Example #7 A senate subcommittee consists of ten Democrats and seven Republicans.  In how many ways can a chairman, vice chairman, and secretary be chosen if

a) there are no restrictions?

b) the chairman must be a Democrat and the vice chairman must be a Republican?

**Day 1 Classwork**

1. An ice cream parlor offers 12 different flavors of ice cream. There are four choices for the container (cup, regular cone, sugar cone, and waffle cone).

2. A computer password consists of four letters followed by a single digit. Assume that the passwords are not case sensitive (i.e., that an uppercase letter is the same as a lower case letter).

a) How many different passwords are possible?

b) How many different passwords end in 1?

c) How many different passwords do not start with Z?

d) How many different passwords have no Z's in them?

3. A French restaurant offers a menu consisting of 3 different appetizers, 2 different soups, 4 different salads, 9 different main courses, and 5 different desserts.

a) A fixed-price lunch meal consists of a choice of appetizer, salad, and main course. How many different lunch fixed-price meals are possible?

b) A fixed-price dinner meal consists of a choice of appetizer, a choice of soup or salad, a main course, and a dessert. How many different dinner fixed-price meals are possible?

4. A set of reference books consists of 8 volumes numbered 1 through 8. In how many ways can the 8 books be arranged on a shelf?

5. Four women and 4 men line up a checkout stand in a grocery store.

a) In how many ways can they line up?

b) In how many ways can they line up if the first person in line must be a woman?

c) In how many ways can they line up alternately by gender?

6. How many 7-digit numbers (i.e., numbers between 1,000,000 and 9,999,999)

a) are even?

b) are divisible by 5?

7. The ski club at East Carolina University has 35 members (15 girls and 20 boys). A committee of 3 members – a President, a Vice-President, and a Treasurer – must be chosen.

a) How many different 3-member committees can be chosen?

b) How many different 3-member committees can be chosen if the president must be a girl?

*c) Bonus: How many different 3-member committees can be chosen if the committee can *not* have all boys or all girls?

Day 2 Permutations and Combinations

A ***permutation*** is an arrangement of a group of objects in a particular order. ________________________*!*

The four numbers 5, 6, 7, and 8 can be arranged in twenty-four different ways:

| | | | | | |
|---|---|---|---|---|---|
| 5,6,7,8 | 5,6,8,7 | 5,7,6,8 | 5,7,8,6 | 5,8,6,7 | 5,8,7,6 |
| 6,7,8,5 | 6,7,5,8 | 6,8,5,7 | 6,8,7,5 | 6,5,7,8 | 6,5,8,7 |
| 7,5,6,8 | 7,5,8,6 | 7,6,5,8 | 7,6,8,5 | 7,8,5,6 | 7,8,6,5 |
| 8,5,6,7 | 8,5,7,6 | 8,6,5,7 | 8,6,7,5 | 8,7,5,6 | 8,7,6,5 |

Each of these arrangements is a different permutation. To determine the total number of permutations that can be made from four digits using each one only once, we indicate a space for each digit:

_____ _____ _____ _____

You can choose any one of the four digits to put in the first space. Then you have any of the remaining three digits to go into the second space. There are now two choices for the third space, and the last digit is placed in the fourth space. The **product** of the number of choices for each space is the total number of permutations that can be made. So we have ________________________________________ different ways to fill the spaces.

**Remember:** Factorial (!) is defined as the product of each subsequent number until you reach 1

9! ____________________ 7!____________________

**Example:** If we were arranging 6 objects we would have ____________________ different ways to fill the spaces.
Any arrangement of *n* objects in a particular order is called a permutation of *n* objects.

We see that there are 24 permutations of 4 different objects and there are 720 permutations of 6 objects. The total number of permutations that can be formed from *n* objects using all of them without repetition is *n*! The symbol *n*! is read *n factorial*. Notice that $_nP_n = n!$ because we are arranging **all** *n* objects.

**What if you don't arrange all the items?**

You are given a set of 6 students who are members of a school club. From these 6 students you have to choose a president, vice-president, secretary and treasurer for the club. In how many ways can you do this? *(Notice that it makes a difference in the way that we choose them. We could choose the same four students but they could serve in different offices.)*

The symbol $_nP_k$ represents the number of permutations that can be formed from *n* objects taken *k* at a time where $k<n$. The number of permutations of six objects taken four at a time is

***Calculator:*** ______________________________________________________________

***Examples:***

1. In a scrabble game, Mario drew the letters E,W,L,N,S,F and O. How many permutations of 4 letters are possible?

2. How many permutations are possible of the letters ABCDWXYZ?

3. How many ways can a president, vice president, secretary and treasurer be elected from a club with 15 members?

## Permutations of objects that are <u>not</u> all different (Distinguishable Permutations)

The four numbers 1, 6, 6, 3 can be arranged in 4! ways. However, two of the numbers are the same so several of the arrangements are identical and cannot be **distinguished** from others. Consider the digits in this way: 1, 6, 6*, 3. The following arrangements can be made:

| | | | | | |
|---|---|---|---|---|---|
| 16*63 | 16*36 | 166*3 | 1636* | 136*6 | 1366* |
| 6*631 | 6*613 | 6*136 | 6*163 | 6*316 | 6*361 |
| 516*3 | 6136* | 6316* | 636*1 | 66*13 | 66*31 |
| 316*6 | 3166* | 36*16 | 36*61 | 3616* | 366*1 |

Although we have written down twenty-four permutations, there are only twelve distinct arrangements. The number of distinct permutations of four objects when two are alike may be denoted by:

The number of distinct permutations of $n$ objects of which $p$ are alike, $q$ are alike, *etc.* is:

***Example***: How many different permutations can be made using all the letters of the word Connecticut?

***Example #2***: Find the number of different ways of placing 15 balls in a row given that 4 are red, 3 are yellow, 6 are blue and 2 are black.

## Combinations

A ***combination*** is an arrangement of a group of objects in which ______________________________________***!***

From the numbers 1, 2, 3, six different permutations can be formed. They are 123, 132, 231, 213, 312, 321. When the order of the digits is not considered, **all six of these permutations make up one combination.** The number of combinations of three objects, taken three at a time is one. In general, the number of combinations of $n$ objects taken $n$ at a time is one.
Now, what if we want to look at the number of combinations that can be made from $n$ objects, taking only $r$ at a time is

***Calculator:*** ________________________________________________________________

***Examples:***

1. In how many ways can a committee of four be chosen from ten people?

2. A student must answer 7 of the 10 questions on an exam. In how many ways can he choose the 7 questions?

3. A committee composed of 3 math majors and 4 science majors is to be selected from a group of 20 math majors and 16 science majors. How many different committees can be formed?

## Permutations and Combinations

***Example:*** A committee of seven, consisting of a chairperson, vice-chairperson, secretary and 4 other members, is to be chosen from a class of 20 students. In how many ways can the committee be chosen?

**Determine if the following are permutations or combinations:**

1. Creating an access code for a computer site using any 8 alphabet letters.

2. Determining how many different ways you can elect a Chairman and Co-Chairman of a committee if you have 10 people to choose from.

3. Voting to allow 10 new members to join a club when there are 25 that would like to join.

4. Finding different ways to arrange a line-up for batters on a baseball team.

5. Choosing 3 toppings for a pizza if there are 9 choices.

**Solve:**

6. Suppose that 7 people enter a swim meet. Assuming that there are no ties, in how many ways could the gold, silver, and bronze medals be awarded?

7. A coach must choose how to select his five starters from a team of 12 players. How many different ways can the coach choose the starters?

8. John bought a machine to make fresh juice. He has five different fruits: strawberries, oranges, apples, pineapples, and lemons. If he only uses two fruits, how many different juice drinks can John make?

9. There are 25 people who work in an office together. Five of these people are selected to attend five different conferences. The first person selected will go to a conference in Hawaii, the second will go to New York, the third will go to San Diego, the fourth will go to Atlanta, and the fifth will go to Nashville. How many such selections are possible?

10. One hundred twelve people bought raffle tickets to enter a random drawing for three prizes. How many ways can three names are drawn for first prize, second prize, and third prize?

11. A disc jockey has to choose three songs for the last few minutes of his evening show. If there are nine songs that he feels are appropriate for that time slot, then how many ways can he choose and arrange to play three of those nine songs?

12. There are 25 people who work in an office together. Five of these people are selected to go together to the same conference in Orlando, Florida. How many ways can they choose this team of five people to go to the conference?

1. How many distinguishable arrangements are possible using the letters from the word POTATO?

2. How many ways can five people line up at a checkout counter in a supermarket?

3. A company car that has a seating capacity of six is to be used by six employees who have formed a car pool. If only four of these employees can drive, how many possible seating arrangements are there for the group?

4. Find the number of distinguishable permutations of the letters SWEET.

5. How many different four-letter permutations are there for the letters in the word "minimum"?

6. A political science professor must select 4 students from her class of 12 students for a field trip to state legislature. In how many ways can she do it?

7. The Canyon Crest Academy basketball team is in a tournament with 5 other teams. In how many ways can the teams finish the tournament?

8. A baseball team has nine players. How many different batting orders are possible assuming that every play will be allowed to bat?

9. Find the number of different ways of placing 15 balls in a row given that 4 are red, 3 are yellow, 6 are black, & 2 are blue.

10. Fourteen construction workers are to be assigned to three different tasks. Seven workers are needed for mixing cement, five for laying bricks, & two for carrying bricks. In how many different ways can the workers be assigned to these tasks

11. If the NCAA has applications from 6 universities for hosting its intercollegiate tennis championships in 2008 and 2009, how many ways may they select the hosts for these championships
    a) if they are not both to be held at the same university?

    b) if they may both be held at the same university?

12. There are five finalists in the Mr. Rock Hill pageant. In how many ways may the judges choose a winner and a first runner-up?

13. In a primary election, there are four candidates for mayor, five candidates for city treasurer, and two candidates for county attorney. In how many ways may voters mark their ballots if they vote in all three of the races?

14. In how many ways may can five persons line up to get on a bus?

15. How many permutations are there of the letters in the word “great”?

16. How many distinct permutations are there of the word “statistics”?

17. How many distinct permutations of the word “statistics” begin and end with the letter “s”?

18. A college football team plays 10 games during the season. In how many ways can it end the season with 5 wins, 4 losses, and 1 tie?

19. If eight people eat dinner together, in how many different ways may 3 order chicken, 4 order steak, and 1 order lobster?

20. Suppose a True-False test has 20 questions.
    a) In how many ways may a student mark the test, if each question is answered?

    b) In how many ways may a student mark the test, if 7 questions are marked correctly and 13 incorrectly?

    c) In how many ways may a student mark the test, if 10 questions are marked correctly and 10 incorrectly?

21. Among the seven nominees for two vacancies on the city council are three men and four women. In how many ways may these vacancies be filled
    a) with any two of the nominees?

    b) with any two of the women?

    c) with one of the men and one of the women?

22. Mr. Jones owns 4 pairs of pants, 7 shirts, and 3 sweaters. In how many ways may he choose2 of the pairs of pants, 3 of the shirts, and 1 of the sweaters to pack for a trip?

23. In how many ways may one A, three B’s, two C’s, and one F be distributed among seven students in an AFM class?

## Permutations and Combinations Practice

**State if each scenario involves a permutation or a combination. Then find the number of possibilities.**

1) A group of 35 people are going to run a race. The top three runners earn gold, silver, and bronze medals.

2) Kali and Kim are planning trips to four countries this year. There are 11 countries they would like to visit. One trip will be one week long, another two days, another two weeks, and the other a month.

3) Mofor has homework assignments in four subjects. He only has time to do one of them.

4) A team of 15 field hockey players needs to choose three players to refill the water cooler.

5) The student body of 60 students wants to elect a president, vice president, secretary, and treasurer.

6) There are 160 students at a meeting. They each shake hands with everyone else. How many handshakes were there?

7) A team of 15 field hockey players needs to choose a captain and co-captain.

8) You are setting the combination on a four-digit lock. You want to use the numbers 1234 but don't care what order they are in.

9) There are 40 applicants for two jobs: computer programmer and software tester.

10) A team of 9 dodgeball players needs to choose two players to refill the water cooler.

11) The student body of 100 students wants to elect a president, vice president, and secretary.

12) Selecting which eight players will be in the batting order on a 12 person team.

13) A group of 25 people are going to run a race. The top three runners earn gold, silver, and bronze medals.

14) 3 out of 11 students will ride in a car instead of a van

15) Mike and John are planning trips to two countries this year. There are 11 countries they would like to visit. One trip will be one week long and the other two weeks.

16) 5 out of 9 students will ride in a car instead of a van

17) There are 10 applicants for four jobs: Computer Programmer, Software Tester, Manager, and Systems Engineer.

18) The batting order for seven players on a 9 person team.

1. How many three letter words can be formed if repetition of the letters is
   (a) allowed?
   (b) not allowed?

2. Dancing with the Stars has 10 female and 6 males who have made it passed round one of the auditions. If the judges want to see them pair up female/male for the next round of elimination, how many dancing pairs can they make?

3. In how many different ways can president, vice president and secretary be chosen from a class of 15 students?

4. In how many different ways can five red balls, two white balls and seven black balls be arranged in a row?

5. In how many ways can six different mathematics books be placed next to each other on a shelf?

6. A true-false test contains 8 questions. In how many different ways can the test be completed?

7. A law firm is creating a committee of 9 members. 3 of the members are to be made up of the 7 senior members, 4 of the members are to be made up of the 6 junior members, and the remainder of the committee is to be made up of the 12 custodians. How many possible committees could be created by this firm?

8. A state has registered 2 million automobiles. To simplify the license plate system, a state employee suggests that each plate display only two digits followed by three letters. Will this system create enough different license plates for all the vehicles registered? Explain why/why not.

9. How many different 6 digit license plates of the same state can have the digits 3, 5, 5, 6, 2, and 6?

10. How many different three-digit whole numbers can be formed using the digits 1, 3, 5 and 7 if no repeating of digits is allowed?

11. There are five women and six men in a group. From this group a committee of 4 is to be chosen. How many different ways can a committee be formed that contain three women and one man?

12. The members of a string quartet composed of two violinists, a violist, and a cellist are to be selected from a group of six violinists, three violists, and two cellists, respectively. In how many ways can the string quartet be formed if one of the violinists is to be designed as the first violinist and the other is to be designated as the second violinist?

13. How many ways are there to seat 4 couples on a bench in line at a restaurant?

14. A committee of 9- consisting of a chairman, vice-chairman, secretary and 6 other members-is to be chosen from a class of 30 students. In how many ways can this committee be chosen?

AFM Day 4 Theoretical vs. Experimental Probability

**Definitions:**

| *Experiment* | *Outcomes* |
|---|---|
| *Sample space* | *Event* |

Let's correlate the definitions with the activity you just did!

To find a ***theoretical probability***, first list all possible outcomes. Then use the formula:
**P(event) = number of favorable outcomes/total number of outcomes**

$$0 \leq P(Event) \leq 1$$

If the value of P(Event) is:

Closer to 1:
Closer to 0:
Equal to 1:
Equal to 0:

You can collect data through observations or experiments and use the data to state the ***experimental probability***.

With your experiments:

| Coin Toss | *P*(Heads) | *P*(Tails) |
|---|---|---|
| **Experimental Probability** | | |
| **Theoretical Probability** | | |

Ask yourself some questions regarding the experiment. Were the results of each experiment what you expected? What would you expect if we were to repeat the process 1000 times? 10,000 times? 100,000 times?

***Law of Large Numbers:*** As your number of outcomes reaches ______________________________ your experimental probability will near the theoretical probability.

**Examples of Theoretical Probability:**

1. An experiment consists of tossing a coin three times and recording the results in order.
   a) What is the sample space?

   b) What is the subset, E, of an event showing "exactly two heads"?

   c) What is the subset, F, of an event showing "at least two heads"?

2. Use same experiment from #1.
   a) What is the probability of getting exactly two heads?

   b) What is the probability of getting at least two heads?

   c) What is the probability of getting no heads?

3. A five-card poker hand is drawn from a deck of 53 cards. What is the probability that all five cards are spades?
   a) What is the sample space?

   b) How many ways can five spades be chosen?

   c) What is the probability that all five cards are spades?

4. A bucket contains 10 red balls and 15 green balls. Six balls are drawn at random from the bucket.What is the probability that at least one ball is red?
   a) What is the sample space?

   b) What are the ways of choosing no red balls?

   c) Probability that at least one ball is red?

## Practice

1. A letter is selected at random from the letters of the word FLORIDA. What is the probability that the letter is an A? The letter is a vowel?

2. Find the probability of getting a heart if you are dealt one card from a standard 52 card deck.

3. Find the probability of getting a number less than 5 when you roll a die once.

4. Eight horses are entered in a race. You randomly predict a particular order for the horses to complete the race. What is the probability that your prediction is correct?

5. A bag contains 20 tennis balls, of which four are defective. If two balls are selected at random from the bag, what is the probability that both are defective?

## Extra Practice

Suppose you have a bag with 75 marbles: 15 red, 5 white, 25 green, 20 black, and 10 blue. You draw a marble, note its color, and then put it back. You do this 75 times with these results: 12 red, 9 white, 27 green, 17 black, and 10 blue. Write each probability as a fraction in simplest form.

| Drawing Marbles | 1. $P$(red) | 2. $P$(white) | 3. $P$(green) | 4. $P$(black) | 5. $P$(blue) |
|---|---|---|---|---|---|
| Experimental Probability | | | | | |
| Theoretical Probability | | | | | |

## Replacement/Non-Replacement

**Think about this...**
Suppose you have 2 bags with the exact same sets of marbles inside. Let's say there are 4 red, 5 blue and 9 yellow marbles in each bag.
From the first bag, you reach in and make a selection. You record the color <u>and then drop the marble back into the bag</u>. You then repeat the experiment a second time.
From the second bag you do exactly the same thing EXCEPT, after you select the first marble and record it's color, <u>you do NOT put the marble back into the bag</u>, You then select a second marble, just like the other experiment.

The first experiment involves a process called
*"with replacement"*.

The second experiment involves a process called
*"without replacement"*.

As you might imagine, the probabilities for the 2 experiments will not be the same. In this lesson we will illustrate a variety of these types of problems and explain how to arrive at the correct solutions.

Examples:

1. A player is dealt 2 cards from a standard deck of 52 cards. What is the probability of getting a pair of aces?
2. A jar contains 2 red and 5 green marbles. A marble is drawn, it's color noted, and put back in the jar. This process is repeated a total of 4 times. What is the probability that you selected 4 green marbles?
3. A bag of candy contains 4 lemon flavored sour balls, and 5 lime flavored sour balls. If Tim reaches in, takes one out and eats it, and then 20 minutes later selects another and eats that one as well, what is the probability that they were both lemon flavored candies?
4. Mary has 4 dimes, 3 quarters and 7 nickels in her purse. She reaches in and pulls out a coin, only to have it slip from her fingers and fall back into the purse. She then picks out another coin. What is the probability that she picked a nickel on both tries?
5. A bag of candy contains 4 lemon flavored sour balls, and 5 lime flavored sour balls. If Tim reaches in, takes one out and eats it, and then 20 minutes later selects another and eats that one as well, what is the probability that the first was lemon and the second was lime?

## Experimental and Theoretical Probability Practice

1. An experiment consists of tossing a coin and rolling a die.
   (a) Find the sample space
   (b) Find the probability of getting heads and an even number
   (c) Find the probability of getting heads and a number greater than 4
   (d) Find the probability of getting tails and an odd number.\

2. A card is drawn randomly from a standard 52-card deck. Find the probability of the given event:
   (a) The card is a heart
   (b) The card is a heart or a spade.
   (c) The card is a heart, spade or diamond.

3. A child's game consists of a spinner as shown in the figure. Find the probability of the given event.
   (a) The spinner stops on an even number
   (b) The spinner stops on an odd number or a number greater than 3


4. A poker hand, consisting of five cards, is dealt from a standard deck of 52 cards. Find the probability that the hand contains:
   (a) 5 hearts
   (b) 5 face cards

5. A pair of dice is rolled and the numbers showing are observed.
   (a) List the sample space of this experiment
   (b) Find the probability of getting a sum of 7
   (c) Find the probability of getting a sum of 9
   (d) Find the probability that the two dice show doubles.

6. What is the probability that a 13-card bridge hand consists of all cards from the same suit?

7. A toddler has wooden blocks showing the letters C, E, F, H, N and R. Find the probability that the child arranges the letters in the indicated order.
   (a) In the order FRENCH
   (b) In alphabetical order

8. The president of a large company selects six employees to receive a special bonus. He claims that the six employees are chosen randomly from among the 30 employees, of which 19 are women and 11 are men. What is the probability that no woman is chosen?

9. Eight horses are entered in a race. You randomly predict a particular order for the horses to complete the race. What is the probability that your prediction is correct?

## Probability Practice 2

1. A coin and a die are tossed. Calculate the probability of getting tails and a 5.
2. In Tania's homeroom class, 9% of the students were born in March and 40% of the students have a blood type of O+. What is the probability of a student chosen at random from Tania's homeroom class being born in March and having a blood type of O+?
3. If a baseball player gets a hit in 31% of his at-bats, what it the probability that the baseball player will get a hit in 5 at-bats in a row?
4. What is the probability of tossing 2 coins one after the other and getting 1 head and 1 tail?
5. 2 cards are chosen from a deck of cards. The first card is replaced before choosing the second card. What is the probability that they both will be clubs?
6. 2 cards are chosen from a deck of cards. The first card is replaced before choosing the second card. What is the probability that they both will be face cards?
7. If the probability of receiving at least 1 piece of mail on any particular day is 22%, what is the probability of *not* receiving any mail for 3 days in a row?
8. Jonathan is rolling 2 dice and needs to roll an 11 to win the game he is playing. What is the probability that Jonathan wins the game?
9. Thomas bought a bag of jelly beans that contained 10 red jelly beans, 15 blue jelly beans, and 12 green jelly beans. What is the probability of Thomas reaching into the bag and pulling out a blue or green jelly bean and then reaching in again and pulling out a red jelly bean? Assume that the first jelly bean is not replaced.
10. For question 10, what if the order was reversed? In other words, what is the probability of Thomas reaching into the bag and pulling out a red jelly bean and then reaching in again and pulling out a blue or green jelly bean *without replacement*?
11. What is the probability of drawing 2 face cards one after the other from a standard deck of cards *without replacement*?
12. There are 3 quarters, 7 dimes, 13 nickels, and 27 pennies in Jonah's piggy bank. If Jonah chooses 2 of the coins at random one after the other, what is the probability that the first coin chosen is a nickel and the second coin chosen is a quarter? Assume that the first coin is not replaced.
13. Jenny bought a half-dozen doughnuts, and she plans to randomly select 1 doughnut each morning and eat it for breakfast until all the doughnuts are gone. If there are 3 glazed, 1 jelly, and 2 plain doughnuts, what is the probability that the last doughnut Jenny eats is a jelly doughnut?
14. Steve will draw 2 cards one after the other from a standard deck of cards *without replacement*. What is the probability that his 2 cards will consist of a heart and a diamond?

## 13.3 Mutually Inclusive and Exclusive

**Two events are mutually exclusive ______________________________________.**

Ex: In drawing a card form a standard deck

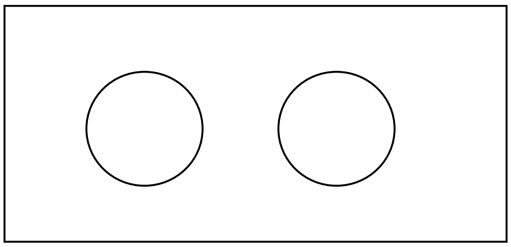
E: the card is an ace

F: the card is a queen

*NOT possible to be an ace and a queen at the same time!

**Probability of the union of <u>mutually exclusive event</u>:**

If E and F are mutually exclusive events in the same sample space S, the probability of E or F is:

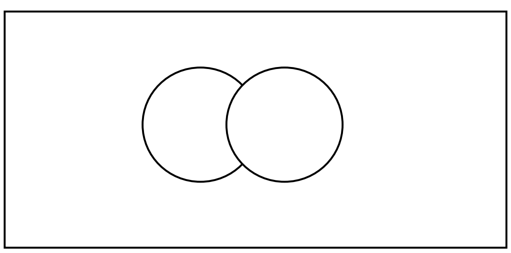


** They cannot happen at the same time!

**Example**: A marble is selected from a bag containing 5 blue, 2 red, and 3 white marbles. What is the probability that it is a blue or a white marble? (a marble cannot be blue and white so mutually exclusive)

**Probability of the union of two events (NOT mutually exclusive)**

If E and F are in the sample space S, then the probability of E <u>or</u> F is



*Think about a venn diagram

**Example**: When drawing a card from a deck of 52 playing cards, what is the probability of getting a red card or a King? (a card can be red and a king at the same time so NOT mutually exclusive)

$P(Red \text{ or } King) = P(Red) + P(King) - P(Red \cap King)$

When the occurrence of one event does not affect the probability of another event, we say the events are ______________________.

Ex: If a balanced coin is tossed, the probability of showing heads on the $2^{nd}$ toss is ½ regardless of the outcome of the $1^{st}$ toss. So any 2 tosses of a coin are independent.

Probability of the intersection of independent events:

If E and F are independent events in a sample space S, then:

(Probability of E times probability of F)

**Example**: When tossing a fair coin twice, what is the probability of getting a 'Head' on the first toss and then getting a 'Tail' on the second toss?
$P(H \text{ and } T) = P(H) \times P(T)$

When the occurrence of one event <u>does</u> affect the probability of another event we say that the events are _____________________.

*We use $P(E \cap F) = P(E) * P(F)$ for dependent events also, but we need to consider the effect on the first event on the second event when considering the sample space.

**Example**: Paul draws a red marble from a bag of 3 red marbles and 2 blue marbles. Then Nadia draws a blue marble from those remaining in the bag. What is the probability of this happening?

**How to identify Probability Problems:**

-If there is an ____________ then it is _______________ problem and we must use _______________.
- You must determine if the two events are mutually exclusive to determine which addition formula.

YES:
NO:

-If there is an ________________ then we are dealing with _______________________ and we will use the multiplication formula.

Replace= _______________________
No replacement= ____________ and you must account for a smaller sample space in the second event.

**Example:** A card is chosen at random from a standard deck of 52 playing cards. Without replacing it, a second card is chosen. What is the probability that the first card chosen is a queen and the second card chosen is a jack?

**Practice:**

1. If a committee of three is to be chosen randomly from five males and nine females, what is the probability that the committee is either all male or all female?

2. A box contains a nickel, a penny, and a dime. Find the probability of choosing first a dime and then, without replacing the dime, choosing a penny.

3. There are 3 literature books, 4 algebra books, and 2 biology books on a shelf. If a book is randomly selected, what is the probability of selecting a literature books or an algebra book?

4. A die is rolled. What is the probability of rolling a 5 or a number greater than 3?

5. Find the probability of tossing two number cubes and getting a 3 on each one.

6. Each of the numbers from 1 to 30 is written on a card and placed in a bag. If one card is drawn at random, what is the probability that the number is a multiple of 2 or a multiple of 3?

7. Joe's wallet contains three $1 bills, four $5 bills, and two $10 bills. If he selects three bills in succession, find the probability of selecting a $10 bill, then a $5 bill, and then a $1 bill if the bills are not replaced.

8. A bag contains 5 red, 3 green, 4 blue, and 8 yellow marbles. Find the probability of randomly selecting a green marble, and then a yellow marble if the first marble is replaced

9. In Jerry's Boy Scout group of 24 scouts, 6 scouts are studying for the swimming merit badge and 10 scouts are studying for the life saving merit badge. Two scouts are studying for both merit badges. Find the probability that a scout is studying for:
   a. Swimming merit badge
   b. Life saving merit badge
   c. Swimming and life saving merit badges
   d. 4. Swimming or life saving merit badge

**All Types of Probability Practice**

1. A hand of 5 cards is dealt. What is the probability…
   a. That you have 4 aces and any other card
   b. 4 tens and an ace
   c. 10, Jack, Queen, King and Ace?

2. A bag contains 26 tiles with a letter on each, one tile for each letter of the alphabet. What is the probability of reaching into the bag and randomly choosing a tile with one of the last 5 letters of the alphabet on it or randomly choosing a tile with a vowel on it?

3. Jack is a student in Bluenose High School. He noticed that a lot of the students in his math class were also in his chemistry class. In fact, of the 60 students in his grade, 28 students were in his math class, 32 students were in his chemistry class, and 15 students were in both his math class and his chemistry class. What is the probability of selecting a student at random who was either in his math class or his chemistry class?

4. One card is selected at random. Find the probability that the card selected is
   a. a face card
   b. a spade or an odd number card
   c. a number card less than 7
   d. a red card or an ace

5. Given a class of 12 girls and 10 boys.
   (a) In how many ways can a committee of five consisting of 3 girls and 2 boys be chosen?

   (b) What is the probability that a committee of five, chosen at random from the class, consists of three girls and two boys?

   (c) How many of the possible committees of five have no boys?

   (d) What is the probability that a committee of five, chosen at random from the class, consists only of girls

6. John and Beth are hoping to be selected from their class of 30 as president and vice-president of the Social Committee. If the three-person committee (president, vice-president, and secretary) is selected at random, what is the probability that John and Beth would be president and vice president?

7. Find the probability of selecting a boy or a blond-haired person from 12 girls, 5 of whom have blond hair, and 15 boys, 6 of whom have blond hair.

8. In a bag there are 2 red marbles, 3 white marbles and 5 blue marbles. Once a marble is selected, it is NOT replaced. Find the following probabilities:
a. P(red, then white) b. P(blue, then red) c. P(red, red, red)

9. Assuming that any date is equally likely to be the birthday of a random person and that there are 365 days in a year, what is the probability that a randomly chosen person's birthday is not on the 31$^{st}$ of a month?

### 13.4 Expected Value and Fairness

The __________________________ is an average expectation per game if the game is played many times!

*The expected return per game

*Payoff= _________________________________

A game gives payoffs of $a_1, a_2, a_3 \ldots a_n$ with the probabilities $p_1, p_2, p_3 \ldots p_n$

The expected value (or expectation) E of this game is:

NOTE! Expected value can be negative! A negative expected value indicates a negative payout (i.e. you're losing money!)

Fairness: Occurs when the __________________ of winning is equally as likely (meaning you have the same chance of winning and losing) or when expected value is such that a player can "break even" (meaning that after playing a game numerous times, his returns will match what he pays to play the game, E=0)

Ex 1: A coin is flipped. Heads, you win $1. Tails, you lose $1. What is the expected value of this game?

*A game whose expected winnings are _______ is called a __________ __________.*

Ex 2: Jane gets $6 if a die shows a 6 and loses $1 otherwise. What is her expectation?

Ex 3: A die is rolled. If the die shows a 1, 2, or 3 you get 10 points. If the die shows a 4 or a 5, you lose 13 points. If the die shows a 6, you lose 1 point. What is the expected value of this game?

Ex 4: In Monte Carlo, the game of roulette is played on a wheel with slots numbered 0, 1, 2, …, 36. The wheel spun and a ball dropped on the wheel is equally likely to end up in any one of the slots. To play the game, you bet $1 on any number. If the ball stops in your slot, you win $36 (the $1 you bet plus $35). Find the expected value of this game.

Ex 5: A sweepstakes contest offers a first prize of one million dollars, a second prize of $200,000, and a third prize of $40,000. Suppose that three million people enter the contest and three names are selected randomly for the three prizes.

(a) What are the expected winnings of a person participating in this contest?

(b) Is it worth paying $0.50 to enter this contest?

Ex 6: Real Life Ex: A life insurance policy for a 40-year old woman will pay $10,000 if she dies within 1 year. The policy costs $300. Statistics (namely, mortality tables) indicate that the relative frequency of a 40-year old woman dying within 1 year is 0.02. What is the expected profit of this policy to the woman?

Ex 7: A game consists of drawing a card from a deck. You win $13 if you draw an ace. What is a “fair price” to pay to play this game? (“Fair price” implies the price at which the player will break even, or in other words, the price at which expectation is zero).

## Expected Value Practice

1. A student plays the following game. He tossed three coins. If he gets exactly two heads he wins \$5. If he gets exactly one head he wins \$3. Otherwise, he loses \$2. On the average, how much should he win or lose per play of the game? (Use the word "win" or "lose" in your answer.

2. A detective figures that he has a $\frac{1}{9}$ chance of recovering some stolen property. He works on a contingency plan. He gets his money if he recovers the property but he does not get his money if he does not recover the property. The investigation costs will be \$9000. How large should his fee be so that, on average, his fee will be covered?

3. At Tucson Raceway Park, your horse, Stick-in-the-mud has a probably of $\frac{1}{20}$ of coming in first place, a probability of $\frac{1}{10}$ of coming in second, and a probability of $\frac{1}{4}$ of coming in third. First place wins \$4500, second place \$3500, and third place \$1500. It costs you \$1000 to enter the race. What is the expected value of the race to you? Is it worthwhile for you to enter the race? Explain.

4. A social club has a drawing every Friday night. The probability of winning the first prize of \$100 is 0.002. The probability of winning the second prize of \$80 is 0.01. How much should the club charge for tickets to enter the drawing so that the club breaks even?

5. You plan to invest in a certain project. There is a 35% chance that you will lose \$30,000, a 40% chance that you will break even, and a 25% chance that you will make \$55,000. What is the expected value in this problem, and what does it mean in terms of your investment?

6. A game consists of tossing a coin twice. A player who tosses two of the same face wins \$1. How much should organizers charge to enter the game if they want to average \$1.00 profit per person?

7. Consider a hat with pieces of paper inside. The papers are numbered as follows: 5 pieces with the number "1," 6 pieces with the number "7," and 9 pieces with the number "50." Find the expected value for drawing from this hat.

8. "Wheel of Fortune" just got a new wheel! On it there are 6 slots worth $200, 15 slots worth $400, 2 slots worth $600, 1 slot worth $1000, 6 slots with no money, and 1 slot with a car worth $20,000. What is the expected winnings on one turn(cash and prizes)?

9. In a game, you roll a die. If you get a 1 or a 5, you would win $5. If you roll a 4 you win $15 and if you roll a 2, 3, or 6 you lose $10. What is the expected value of one roll of the die?

10. A raffle is held by the MSUM student association to draw for a $1000 plasma television. Two thousand tickets are sold at $1.00 each. Find the expected value of one ticket.

11. A game consists of rolling a colored die with three red sides, two green sides, and one blue side. A roll of a red loses. A roll of green pays $2.00. A roll of blue pays $5.00. The charge to play the game is $2.00. Would you play the game? Why or why not?

12. A company believes it has a 40% chance of being successful on bidding a contract that yields a profit of $30,000. Assume it costs $5,000 in consultant fees to prepare the bid. What is the expected gain or loss for the company if it decides to bid on the contract?

13. A department store wants to sell eight purses that cost the store $40 each and 32 purses that cost the store $10 each. If all purses are wrapped in forty identical boxes and if each customer picks a box randomly, find
    (a) each customer's expected value if a customer pays $15 for a box

    (b) the department store's total expected profit (or loss) during this sale.

14. Assume that the odds against a certain horse winning a race are 5 to 2. If a better wins $14 when the horse wins, how much should the person bet to make the game "fair"?

## Geometric Probability

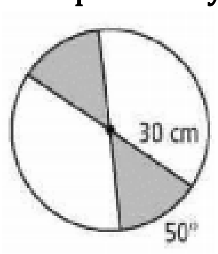
**Theoretical Geometric Probability:** Ratio of successful outcomes to possible outcomes related to areas and lengths.
Remember that probability is the ratio of successful outcomes to possible outcomes.

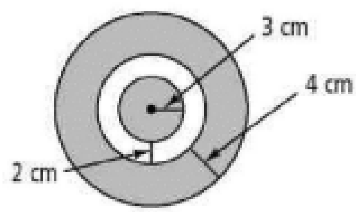
$$\boldsymbol{Probability = \frac{Area\ of\ Desired\ Region}{Total\ Area}}$$

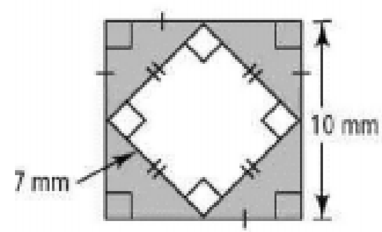
| Circle: | Square/Rectangle: | Sector of a Circle: | Triangle: |
|---|---|---|---|
| $A = \pi r^2$ | $A = l \cdot w$ | $A = \frac{\theta}{360} \cdot \pi r^2$ | $A = \frac{1}{2}bh$ or $A = \frac{1}{2}absinC$ |

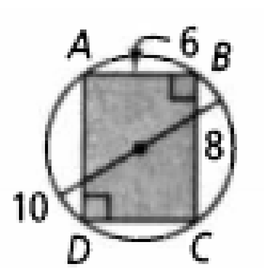
Example: A desk is 29 inches wide and 60 inches long. On it is a desk pad that is 17 inches wide and 22 inches long. If a person randomly flips a paperclip on the desk, what it is the probability that it will land on the desk pad?

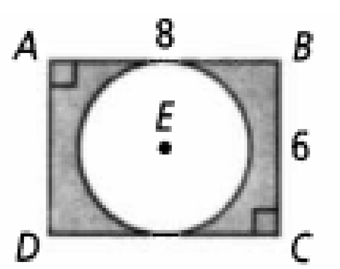
**Find the probability that a point chosen at random will land in the shaded area.**

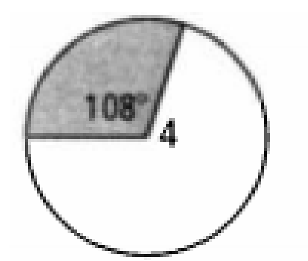
1. 


2. 


3. 


4. 


5. 


6. 

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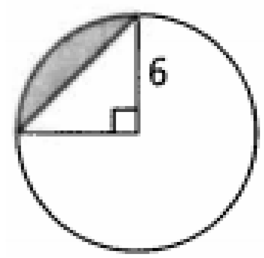


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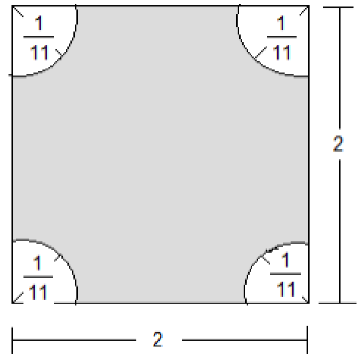


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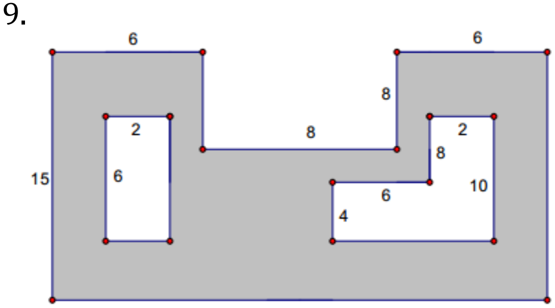
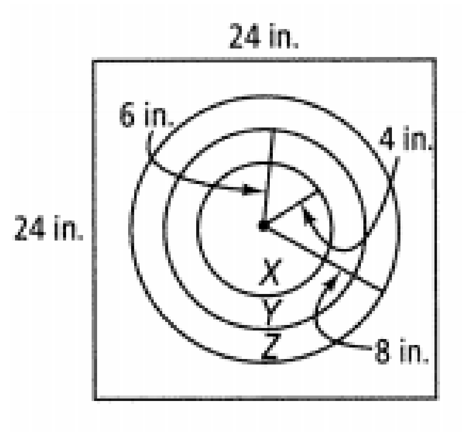



10. a. What is the probability of hitting region X?

b. What is the probability of hitting region Y?

c. What is the probability of hitting region Z?

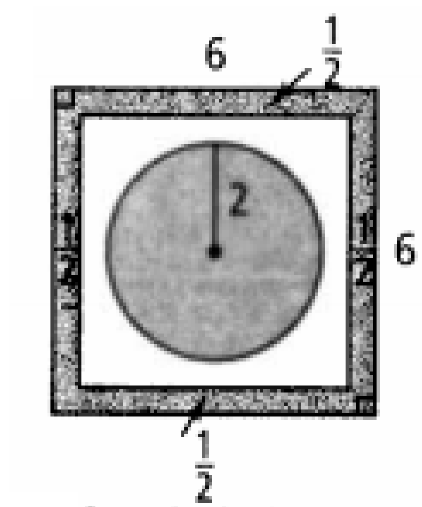


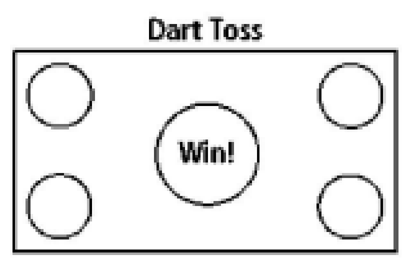

11. You and your friend are playing a target game based on the board to the right. You must hit the border to win a point. Your friend must hit the circle in the center.

a. Is this game fair? That is, do you or your friend have an equal probability of hitting your target zone? Explain.

b. Find the radius of the circle that would make this game fair.

c. Find the probability that you do not score a point.




12. In the fundraiser game at the right, players toss darts at a board to try to get them into one of the holes. The diameter of the center hole is 8 in. The diameter of each of the four corner holes is 5 in. The board is a 20 in. by 30 in. rectangle. Find the probability that a tossed dart will go through the indicated hole.

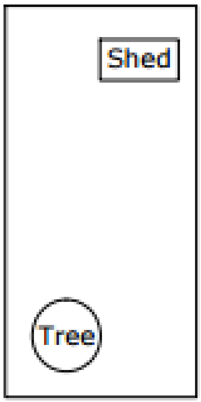
a. center hole

b. top right or left corner

c. any corner

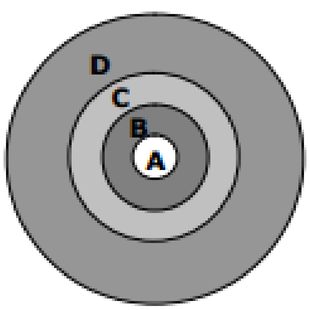
# Geometric Probability Practice

**Round all probabilities to the nearest tenth of a percent.**




1. A rectangular field measures 27 feet by 15 feet. Find the area of the field.
2. A small shed is on the field. Its dimensions are 8 feet by 10 feet. What is its area?
3. What is the probability that a single drop of rain that lands in the filed would hit the shed?
4. What is the probability that a single drop of rain that lands in the field would *not* hit the shed?
5. There is a large oak tree in one corner whose branches have a diameter of 20 feet. What is the probability that a single drop of rain that lands in the field would miss both the shed an the tree? (Assume the shed is not under the tree.)

**A dartboard is made up of concentric circles with the following radii:**
**Circle A: r = 2 inches**
**Circle B: r = 4 inches**
**Circle C: r = 6 inches**
**Circle D: r = 10 inches**




6. Find the area of circle *A*.
7. Find the area of circle *B* that is *not* covered by circle *A*.
8. Find the area circle *C* that is *not* covered by circle *A* or *B*.
9. Find the area of the dartboard that is *not* covered by circles *A*, *B*, or *C*.

**The circles on the dartboard are painted on a rectangular piece of corkboard that is 2 feet by 30 inches. Find the probability of each event, assuming the dart always lands on the corkboard.**

10. A random dart lands on one of the circles.
11. A random dart lands on circle *C* or *D*.
12. A random dart will make a bull's-eye.
13. A random dart falls only on circle *C*.

**Binomial Probability:**

If the probability of <u>success</u> is $p$, the probability of <u>failure</u> is ____________.
Such an experiment whose outcome is random and can be either of two possibilities, "success" or "failure", is called a _________________, after Swiss mathematician Jacob Bernoulli (1654 - 1705).

**Examples** of Bernoulli trials:

- flipping a coin -- heads is success, tails is failure
- rolling a die -- 3 is success, anything else is failure
- voting -- votes for candidate A is success, anything else is failure
- determining eye color -- green eyes is success, anything else is failure
- spraying crops -- the insects are killed is success, anything else is failure

When computing a **binomial probability**, it is necessary to calculate and multiply three separate factors:
1.
2.
3.

| The probability of an event, p, occurring exactly r times |
| --- |
| $${}_nC_r \bullet p^r \bullet q^{n-r}$$ $n$ = number of trials<br>$r$ = number of specific events you wish to obtain<br>$p$ = probability that the event will occur<br>$q$ = probability that the event will **not** occur ($q$ = 1 - $p$, the *complement* of the event) |

**Example:**

1. A test consists of 10 multiple choice questions with five choices for each question. As an experiment, you GUESS on each and every answer without even reading the questions. What is the probability of getting exactly 6 questions correct on this test?

2. At a certain intersection, the light for eastbound traffic is red for 15 seconds, yellow for 5 seconds, and green for 30 seconds. Find the probability that out of the next eight eastbound cars that arrive randomly at the light, exactly three will be stopped by a red light.

3. When rolling a die 100 times, what is the probability of rolling a "4" exactly 25 times?

**Note**: When computing **"at least"** and **"at most"** probabilities, it is necessary to consider, in addition to the given probability,

- all probabilities larger than the given probability ("at least")
- all probabilities smaller than the given probability ("at most")

**Example**: A bag contains 6 red Bingo chips, 4 blue Bingo chips, and 7 white Bingo chips.
What is the probability of drawing a red Bingo chip at least 3 out of 5 times? Round answer to the *nearest hundredth*.
To solve this problem, we need to find the probabilities that $r$ could be 3 or 4 or 5, to satisfy the condition "at least".

It will be necessary to compute $\binom{n}{r} \cdot p^r \cdot (1-p)^{n-r}$
for $r = 3$, $r = 4$ and $r = 5$ and add these three probabilities for the final answer.
We need to compute:

| | |
|---|---|
| For $r = 3$: | |
| For $r = 4$: | |
| For $r = 5$: | |
| Sum: | |

**Remember**: “At most 2” is equivalent to “at least 5” when referring to 5 cases.

**Examples:** A family consists of 3 children. What is the probability that at most 2 of the children are boys?

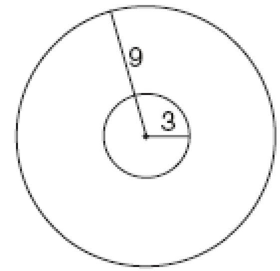
**Solution:**
***"At most" 2 boys implies that there could be 0, 1, or 2 boys. The probability of a boy child (or a girl child) is 1/2.***

| | |
|---|---|
| **For $r = 0$:** | |
| **For $r = 1$:** | |
| **For $r = 2$:** | |
| **Sum:** | |

**2.** Team A and Team B are playing in a league. They will play each other five times. If the probability that team A wins a game is 1/3, what is the probability that team A will win at least three of the five games?
**Solution:**
*"At least" 3 wins implies 3, 4, or 5 wins.*

| | |
|---|---|
| For $r = 3$: | |
| For $r = 4$: | |
| For $r = 5$: | |
| Sum: | |

3. As shown in the accompanying diagram, a circular target with a radius of 9 inches has a bull's-eye that has a radius of 3 inches. If five arrows randomly hit the target, what is the probability that at least four hit the bull's-eye? Express answer to the *nearest thousandth.*




**Solution:**
*"At least" 4 hits implies 4 or 5 hits. The area of the bull's-eye is* $9\pi$ *and the area of the entire target is* $81\pi$. *The probability of hitting the desired bull's-eye is 1/9.*

| | |
|---|---|
| For $r = 4$: | |
| For $r = 5$: | |
| Sum: | |

## Binomial Probability Practice

Given the number of trials and the probability of success, determine the probability indicated: Set up problems 1-4 without the calculator.

1. $n = 12,\ p = 0.2,$ find P(2 successes)

2. $n = 15,\ p = 0.9,$ find P(11 successes)

3. $n = 7,\ p = \frac{1}{3},$ find P(4 successes)

4. $n = 15,\ p = 0.99,$ find P(1 failure)

5. $n = 6,\ p = 0.35,$ find P(at least 3 successes)

6. $n = 100,\ p = 0.01,$ find P(no more than 3 successes)

7. In a history class, Colin and Diana both write a multiple choice quiz. There are 10 questions. Each question has five possible answers. What is the probability that

a) Colin will pass the test if he guesses an answer to each question.

b) Diana will pass the test if she studies so that she has a 75% chance of answering each question correctly.

8. The manufacturing sector contributes 17% of Canada's gross domestic product. A customer orders 50 components from a factory that has a 99% quality production rate (99% of the products are defect-free). Find the probability that:

a) none of the components in the order are defective

b) there is at least one defective product in the order.

c) There are at least two defective products in the order.

9. A pair of dice is rolled 20 times. What is the probability that a sum of 5 is rolled

a) exactly 6 times

b) at least 4 times

c) at most 5 times

10. The probability the Tim will sink a foul shot is 70%. If Tim attempts 30 foul shots, what is the probability that

a) he sinks exactly 21 shots

b) he sinks at most 21 shots

c) he sinks between 18 and 20 shots, inclusive.

# Binomial Probability Practice

1. Compute the probability of X successes, using the binomial formula.
   (a) $n = 5, X = 2, p = 0.025$
   (b) $n = 12, X = 6, p = 0.45$
   (c) $n = 6, X = 0, q = 0.35$
   (d) $n = 45, X = 10, p = 0.25$
   (e) $n = 22, X = 20, p = 0.68$

2. Compute the probability of X successes given $n = 12$ and $p = 0.45$ using the binomial formula.
   (a) $P(X = 6)$
   (b) $P(X \geq 9)$
   (c) $P(X<4)$
   (d) $P(4 <X<7)$
   (e) $P(5<X<7)$

3. A student randomly guesses at 10 multiple choice questions. Each question has four possible answers with only one being correct, and each is independent of every other question.
   (a) Find the probability that the student guesses EXACTLY 4 correct.
   (b) Find the probability of guessing less than 3 correctly.
   (c) Find the probability of guessing more than 8 correctly.
   (d) Find the probability of guessing between 4 and 6 inclusively.

4. In a Gallop Poll conducted January 30 – February 2, 2008, 43% of 18-29 year olds said that they were worried about retirement. Find the probability that out of 15 college students ages 18 – 19:
   (a) Exactly 1 worried about retirement.
   (b) Fewer than 5 worried about retirement.
   (c) At least 10 worried about retirement.
   (d) Between 8 and 10 inclusively are worried about retirement.

5. In a Gallop Poll, 35% of 30-49 year olds stated they believe in ghosts. Find the probability that out of 16 college students aged 30 – 49:
   (a) Exactly 5 said they believed in ghosts.
   (b) Exactly 5 said they do not believe in ghosts.
   (c) At least 4 believe in ghosts.
   (d) At least 4 do not believe in ghosts.

## Probability Review Topics

**Permutation: Order matters**

nPr

**Distinguishable Permutations**

$$\frac{(total\ items)!}{(repeat)!(repeat)!\ldots}$$

**Combination: Order doesn't matter**

nCr

**Fundamental Counting Principle**

*using all items, fill in blanks, multiply

Probability: Likelihood of an event occurring to total number of outcomes

**Independent/Dependent (AND) vs. Mutually Inclusive/Exclusive (OR)**

| AND…MULTIPLY | OR…ADD |
|---|---|
| **Independent**<br>One event does not affect the outcome of the second event<br>Ex: Flipping a coin and rolling a die<br>P(A) x P(B) | **Mutually Exclusive**<br>The events cannot happen at the same time<br>Ex: Being a boy vs being a girl<br><br>P(A)+P(B) |
| **Dependent**<br>One event affects the outcome of the second event<br>Ex> picking a card and picking a second card without replacing the first card<br>P(A) x P(B) (after A happens) | **Mutually Inclusive**<br>The events can happen at the same time<br>Ex: Being a boy and having blue eyes<br><br>P(A)+ P(B) – P(A and B) |

**Binomial Probability**

(exactly, at least, at most)

| The probability of an event, p, occurring exactly r times |
|---|
| ${}_nC_r \bullet p^r \bullet q^{n-r}$<br>$n$ = number of trials<br>$r$ = number of specific events you wish to obtain<br>$p$ = probability that the event will occur<br>$q$ = probability that the event will **not** occur ($q$ = 1 - $p$, the *complement* of the event) |

**Expected Value**

Make a table like this…

| **Probability** | | | |
|---|---|---|---|
| **Outcome** | | | |

E=(outcome)(probability)+(outcome)(probability)+…

*Don't forget the amount you pay to play is a negative *All probability adds up to 1

**Geometric Probability**

$$\boldsymbol{Probability = \frac{Area\ of\ Desired\ Region}{Total\ Area}}$$

Circle: $A = \pi r^2$

Square/Rectangle: $A = l \cdot w$

Triangle: $A = \frac{1}{2}bh$ or $A = \frac{1}{2}absinC$

**Probability Review**

1. Three fair coins are tossed. What is the probability that there is only one head?

2. A poker hand consisting of five cards is dealt from a deck of 52 cards. Find the probability of all five cards being spades.

3. A jar contains 5 red, 4 green balls and 2 yellow balls. Find the probability of the given event.
   a. A green ball is drawn and replaced and then a yellow ball is drawn.

   b. A red ball is drawn and then a white ball is drawn without replacing the firs ball.

   c. A red ball is drawn and then another red ball is drawn without replacing the first ball.

4. A researcher claims that she has taught a monkey to spell the word LEOPARD using six wooden letters D, A, R, L, O, P, and E. If the monkey has not actually learned anything and is merely arranging the blocks randomly, then
   a. What is the probability that he will spell the word correctly

   b. What is the probability that he will spell the word correctly three consecutive times?

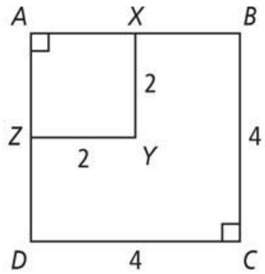
5. A card is drawn, a die is rolled and a coin is tossed. Find the probability of each outcome.
   a. The queen of hearts, a two and a tails,

   b. A face card, a number more than three, and a heads.

6. In our class, what is the probability of picking a student that is male or picking a student that has blonde hair?

7. You are taking a true/false quiz with 9 questions. What is the probability of getting all 9 questions correct?

8. Point P in square ABCD is chosen at random. Find the probability that point P is in squatter AXYZ.




9. The probability of a student passing my class is 0.8. If 5 students are selected at random, what is the probability that at least 4 students will pass the class.

10. At the State Fair there is a booth where people can throw dimes onto a table that has dishes on it. Suppose that the chance that a dime lands in a dish is 0.3. Suppose you play the game 10 times.
    a. What is the probability that you will throw a dime in a dish at least 8 times?

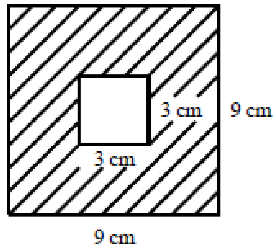
    b. What is the probability that you will throw a dime in the dish exactly 7 times?

11. You won a trip to Vegas and are feeling pretty lucky! You decide to play a game where you roll a die 8 times. What is the probability that
    a. You roll a number greater than 4 at most 2 times?

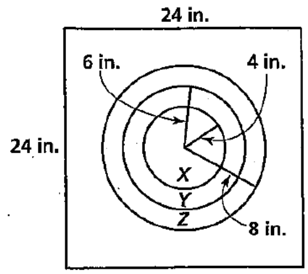
    b. You roll an even number exactly 3 times?

    c. You roll a 6 at least 7 times?

12. Find the probability of landing in the shaded region.



13. Use the following dart board to answer the questions below:




a. If a dart hits the board, find the probability that it will land in region X

b. If a dart hits the board, find the probability that it will land in region Z

c. If a dart hits the board, find the probability that it will NOT hit any of the circles.

14. Three digit numbers are formed using the digits 2, 4, 5 and 8, with repetition of digits allowed. How many such numbers are there that are:
less than 800? even?

15. A fancy restaurant offers 6 appetizers, 4 types of salads, 12 main courses and 3 desserts. In how many different ways can a customer order a meal from this restaurant?

16. Mr. Davis has 24 people, 16 female and 8 male, trying out for the school play. He wants to choose a leading male, a leading female, a supporting male, supporting female and 6 extras - 2 males and 4 females. In how many ways can the cast be chosen?

17. Art, Becky, Carl, Denise, and Ed all want to go see Carrie Underwood in concert. However, they only have 3 tickets. How many ways can they choose the three who get to go to the concert?

18. A volleyball team has nine players. In how many ways can a starting line-up be chosen consisting of two forward players and three defense players?

19. Part I of an exam has 5 multiple choice questions with 4 choices for each question. In how many different ways can this part of the exam be completed?

20. Refer to question 6, assume you randomly guess the answers at the answers. What is the probability that you do not get all answers correct?

21. A child's game has a spinner which has spaces labeled 1 to 9 and all of the spaces are of equal size. What is the probability that the spinner stops on an odd number or a number greater than 6?

22. A bowl contains 5 oranges and 4 tangerines. Noelle randomly selects one, puts it back, and then selects another. What is the probability that both selections are tangerines?

23. A ball is randomly selected from an urn that contains five red balls, three white balls, and one yellow ball. Find the probability that the ball is red or yellow.

24. Two balls are randomly selected from an urn that contains five red balls, three white balls, and one yellow ball. If the ball is not replaced after the first is selected, find the probability that both are red.

25. You pay $0.50 to draw one card from a deck of cards. If it is an ace, you win $10; if it is a face card, you win $1; otherwise, you lose. What is the expected value of this game? Is this game fair?

26. A $1 bet is made to draw three cards from a standard deck of 52 cards. If all three cards are face cards (12 face cards in a deck), then you win $4.00. Find the expectation of this game and explain if you should play or not.

27. Suppose you surveyed the students in your class on their favorite juice flavors. Their choices were 6 apples, 10 orange, 1 grapefruit, and 3 mangos. Record the *experimental* probability for each flavor.