Unit 1: Piecewise Functions

Objective 2.02 Use piece-wise defined functions to model and solve problems; justify results.
a) Solve using tables, graphs and algebraic properties.
b) Interpret the constants, coefficients, and bases in context of the problem.

DAY	TOPIC	ACTIVITY
1	Function: yes/no	
Monday,	Looking at and sketching	
August 24	different graphs of functions	
	to find domain/range and	
	where the graph is	
	increasing/decreasing	
	Interval notation	
2	Domains of functions	
Tuesday,	without looking at the graph	
August 25	Evaluating Functions	
3	Finish domains	
Wednesday,	Intro to parent functions and	
August 26	transformations	
4	Transformations	
Thursday,		
August 27		
5	Intro to Piecewise functions	QUIZ (Days 1-3)
Friday, August		
28		
6	Applications of piecewise	
Monday,	functions	
August 31		
7	Step Functions and	QUIZ (Transformations of
Tuesday,	Applications of piecewise	functions)
September 1	functions continued	
8	Review	
Wednesday,		
September 2		
9	Test	Test
Thursday,		Math XL Due
September 3		

Function:

Domain:

Increasing:

Decreasing:

Range:

Interval Notation:

Parenthesis, brackets or a combination of both.

(): use when there are unknown values

[]: values that you know the function can be equal to

Union "U": The Vertical Line Test:

Examples: Determine if the graph is a function. If it is, state the domain and range.



Graphing Functions:

Asymptotes:

Vertical:

Horizontal:

Open Circle:

Closed Circle:

Examples: Determine the interval on which the function increases or decreases.



Sketch the graph of the following functions. Find the domain, range, and the interval of increase/decrease.



3

Function Classwork Continued

Directions: a)Determine if the graph is a function, b) Find the domain and range of the graph, c) Identify where the graph is increasing/decreasing/constant



Notes: Finding Domain without a Graph

Look for restrictions
Example: $f(x) = \frac{1}{x-4}$ When would this function be undefined?
Example: $f(x) = \sqrt{9 - x^2}$ When would this function be undefined?
Rational functions (fractions) have restrictions
How can we find the restrictions?

Square Root Functions are undefined when _____. How can we find the restrictions?

Examples

1)
$$f(x) = \frac{1}{x^2 - x}$$
 2) $f(x) = \sqrt{9 - x^2}$ 3) $h(t) = \frac{t}{\sqrt{t+1}}$

Domain without Graphs Practice

Practice Find the domain of the functions 1. $f(x) = x^2 + 1$ 8. $f(x) = \frac{x}{\sqrt{x+3}}$ 2. f(x) = 59. $h(x) = \frac{2x+4}{x^2-9}$ 3. $f(x) = \frac{1}{x+1}$ 10. $k(x) = \sqrt{x+4}$ 4. $f(x) = x^3$ $l(x) = \frac{x+3}{\sqrt{x+4}}$ 5. $y(c) = \frac{2}{c^2 + 3c}$ 12. $p(x) = \sqrt{x^2 + 4x - 5}$ 6. $q(w) = \frac{w+4}{w^2+1}$ $g(x) = \frac{x+4}{x-4}$

7. $t(v) = \sqrt{v^2 + 2v - 8}$

14.
$$n(x) = \sqrt{8 - 2x}$$

Evaluating Functions

Remember, a variable is just a place holder. So to evaluate a function *f* at a number, you substitute the number for the placeholder.

Let $f(x) = 3x^2 + x - 5$. Evaluate the function value. (a) f(-2) (c)f(4)(b) f(0) (d) $f(\frac{1}{2})$

Evaluating a function with new variables will require a lot of simplification. If $f(x) = 2x^2 + 3x - 1$, evaluate the following. (a) f(a)

(b) **f**(-**a**)

(c)
$$f(a+h)$$

(d)
$$\frac{f(a+h)-f(a)}{h}, h \neq 0$$

Evaluate the Following Functions

- 1. w(n) = 4n + 2; Find w(3n)
- 2. p(t) = 4t 5; Find p(t 2)
- 3. w(a) = a + 3; Find w(a + 4)
- 4. h(x) = 4x 2; Find h(x + 2)

- 5. $g(n) = n^3 5n^2$; Find g(-4n)
- 6. $f(n) = n^2 2n$; Find $f(n^2)$
- 7. $p(a) = a^3 5$; Find p(x 4)

I. Functions

Graph each of the following functions. Use the table to help you come up with quality graphs! Label at least 5 points on your graph.



- These are called ______ functions or ______ functions. There are sever other types but we will use these for our examples.
- We will investigate how we can ______ the _____ function to get a new graph.
- A transformation is a _____ in a graph from the parent function
- ______ transformation ______ the graph, but doesn't stretch or shrink the shape.

Transformation Discovery

Part 1: Graph the following on your calculator and write down the relationship between the toolkit graph and its transformation. What do you do to the original to get the "new" graph?

Original #1: $y = x^2$ Original #2: y = |x|a) $y = (x + 1)^2$ b) $y = (x - 4)^2$ c) y = |x + 2|d) y = |x - 5|

e) In one complete sentence, predict what the graph of $y = (x + 6)^3$ will look like.

f) In one complete sentence, predict what the graph of $y = \sqrt{x-3}$ will look like.

<u>Generalize</u>: What transformation occurs to the graph of y = f(x) if you have y = f(x + c)?

What transformation occurs to the graph of y = f(x) if you have y = f(x - c)?

Part 2: Graph the following on your calculator and write down the relationship between the toolkit graph and its transformation. What do you do to the original to get the "new" graph?

Original #1: $y = x^2$ Original #2: y = |x|a) $y = x^2 + 1$ b) $y = x^2 - 4$ c) y = |x| + 2d) y = |x| - 5

e) In one complete sentence, predict what the graph of $y = x^3 + 6$ will look like.

f) In one complete sentence, predict what the graph of $y = \sqrt{x} - 3$ will look like.

<u>Generalize</u>: What transformation occurs to the graph of y = f(x) if you have y = f(x) + c?

What transformation occurs to the graph of y = f(x) if you have y = f(x) - c?

Part 3: Graph the following on your calculator and write down the relationship between the toolkit graph and its transformation. What do you do to the original to get the "new" graph? Specifically, look at the 3 original points compared to those 3 "new" points.

<u>Original #1:</u> $y = x^2$	(0,0); (1, 1); (2, 4)	<u>Original #2:</u> $y = x $	(0,0); (-1, 1); (2, 2)
a) $y = 4x^2$	b) $y = \frac{1}{4}x^2$	c) $y = 2 x $	d) $y = - x $

e) The points (0,0), (1,1) and (4,2) are on the graph of $y = \sqrt{x}$. What corresponding points do you think will be on the graph of $y = 4\sqrt{x}$?

<u>Generalize</u>: What transformation occurs to the graph of y = f(x) if you have y = kf(x) where k is positive?

What transformation occurs to the graph of y = f(x) if you have y = -f(x)?

Part 4: Look at the graph of $y = \sqrt{x}$ and specifically the points (1,1), (4, 2) and (9, 3).

a) Graph $y = \sqrt{2x}$. Fill in the blanks in these ordered pairs: (___, 1); (___, 2); (___, 3)

b) Now graph $y = \sqrt{4x}$. Fill in the blanks in these ordered pairs: (___, 1); (___, 2); (___, 3)

- c) Now graph $y = \sqrt{\frac{1}{2}x}$. Fill in the blanks in these ordered pairs: (___, 1); (___, 2); (___, 3)
- **Generalize**: What transformation occurs to the graph of y = f(x) if you have y = f(kx) where k is positive?

Part 5: Look at the graph of $y = \sqrt{-x}$. Describe the difference between it and the graph of $y = \sqrt{x}$.

a) Compare $y = x^3$ and $y = (-x)^3$. Describe the difference. What happened to the original to get the new graph?

<u>Generalize</u>: What transformation occurs to the graph of y = f(x) if you have y = f(-x)?

Part 6: Put it all together! Without using your calculator, describe the transformations of the graphs. Without graphing, how will the following functions be changed from the parent graph?

a) f(x) = -|x + 1| + 1b) $g(x) = -\sqrt{x + 2} - 3$ c) $q(x) = 2x^2 + 4$ d) $r(x) = 2 - \frac{1}{2}(x - 3)^2$

Part 7: Think outside the box.

a) Predict what will happen to the graph of $y = \sqrt{-x + 1}$. Do not use your calculator yet!

b) Now, using your calculator graph $y = \sqrt{-x + 1}$. What happened to the graph? Were you correct?

c) Why do you think the rules applied differently to the graph?

Transformation Rules

<u></u>	<u>ormation raiob</u>
Horizontal Transformations:	Reflections
F(x+c)	-F(X)
F(x-c)	F(-x)
Vertical Transformations	Stretch/Shrink
F(x) + C	a*F(x)
F(x) – C	1/a*F(x)
	F(a*x)
	$F(1/a^*x)$

Transformations of Functions Practice

1. Given the function $f(x) = x^2$, write the function whose graph of f(x) is:

- A. shifted 6 units to the left
- B. reflected about the y-axis
- C. reflected about the x-axis
- D. shifted 5 units up
- E. vertically stretched by a factor of 4
- F. horizontally stretched (compressed) by a factor of 1/3

2. Given the function f(x) = |x|, write the function whose graph of f(x) is:

- A. shifted 6 units to the left
- B. reflected about the y-axis
- C. reflected about the x-axis
- D. shifted 5 units up
- E. vertically stretched by a factor of 4
- F. horizontally stretched (compressed) by a factor of 1/3
- 3. Write a function that is obtained after the following transformations are applied to y = |x|. A. shift 2 units up, reflect about the x-axis then about the y-axis.

B. reflect over the x-axis, shift 3 units left and 2 units up.

4. Consider the following function f(x):

- A. Graph f(x 3)B. Graph f(-x)
- C. Graph -f(x)



Transformation of Functions Practice 2

Draw the parent graph first using a dotted (---)line. Then sketch the final graph using a solid (----) line. Describe each transformation and identify the domain and range of the final graph.

 $1.f(x) = 2x^2 + 3$ Parent Function: Transformation:_____ -4 -2 0 2 4 6 8 10 Domain:_____ Range:_____ 2. f(x) = 3 - |x + 1|Parent Function: _____ Transformation:_____ Domain:_____ Range: $3.h(x) = \frac{1}{2}\sqrt{-x+3}$ Parent Function: Transformation: Domain:_____ Range:_____ $4.f(x) = -(x^3 + 1) - 2$ Parent Function: _____ Transformation: Domain:_____ 0 -8 -6 -4 -2 Range:_____ 5. $g(x) = \frac{1}{3}x^2 - 4$ Parent Function: Transformation:_____ Domain:_____ Range:_____ 6. $y = 3\sqrt{4+x} - 1$ Parent Function: Transformation: Domain:_____ Range:_____

Quiz Review

State the domain and range, where the graph is increasing, decreasing and constant in interval notation.



- A. Find the domain of each of the following functions.
- 5. $f(x) = x^2 3$
- 8. $g(x) = \sqrt{3x 7}$ 6. $f(x) = \frac{1}{x^2 - 6x + 9}$

9. f(x) = |x + 4|

7. $f(x) = \frac{3x}{x^2 - 8x + 15}$ $10. f(x) = \frac{\sqrt{x+1}}{x-7}$

Evaluate the following functions:

11. If
$$f(x) = x^3 + 2$$
, find:
a) f(-2) b) f(3) c) f(0) d) f(a)

12. Find
$$\frac{f(a-h)-f(a)}{h}$$
, $h \neq 0$ for $f(x) = -2x^2 + 1$

Piecewise Functions:

Continuous:

Discontinuous:

Evaluating Piecewise Functions

Evaluate the following function at x = -2, 1, 2, and 3 $f(x) = \begin{cases} 1 - x, if \ x \le 1 \\ x^2, if \ x > 1 \end{cases}$

Evaluate the following functions

1. (a) Find the domain and range of

the graph

(b) Find the values for h(-2), h(0),

h(-3)

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2. (a) Find the domain and range of the graph

(b) Find the values for h(-1), h(1), h(2)



3. (a) Find the domain and range of the graph
(b) Find the values for h(-5), h(-2), h(2), h(4)



Graphing Piecewise Functions

Both of the following notations can be used to describe a piecewise function over the function's domain:

$$f(x) = \begin{cases} 2x & if[-5,2) \\ 5 & if[2,6] \end{cases} \text{ or } f(x) = \begin{cases} 2x & ,-5 \le x < 2 \\ 5 & ,2 \le x \le 6 \end{cases}$$

3. Complete the following table of values for the piecewise function over the given domain.

Х	f(x)
-5	
-3	
0	
1	
1.7	
1.9	
2	
2.2	
4	
6	



- 4. Graph the ordered pairs from your table to Sketch the graph of the piecewise function.
- 5. How many pieces does your graph have? Why?
- 6. Are the pieces rays or segments? Why?
- 7. Are all the endpoints solid dots or open dots or some of each? Why?
- 8. Were all these x values necessary to graph this piecewise function, or could this have been graphed using less points?
- 9. Which x values were "critical" to include in order to sketch the graph of this piecewise function?

- 10. Can you generalize which x-values are essential to input into your table to make a hand sketched graph of a piecewise linear function?
- 11. Now graph this piecewise function: $f(x) = \begin{cases} x+3 & ,-8 \le x < 1\\ 10-2x & ,1 \le x \le 7 \end{cases}$

by completing a table of values for the piecewise function over the given domain.

Х	f(x)

- 12. Why did you choose the x values you placed into the table?
- 13. Graph the ordered pairs from your table to Sketch the graph of the piecewise function.



- 14. How many pieces does your graph have? Why?
- 15. Are the pieces rays or segments? Why?
- 16. Are all the endpoints filled circles or open circles or some of each? Why?
- 17. Was it necessary to evaluate both pieces of the function for the x-value 1? Why or why not?
- 18. Which x values were "critical" to include in order to graph this piecewise function? Explain.

Lesson 2: Writing piecewise functions given a graph.

- 19. Can you identify the equations of the lines that contain each segment?
 - a. Left segment equation=
 - b. Middle equation=
 - c. Right equation=
- 20. Next, list the domain of each segment.
 - a. Left segment domain=
 - b. Middle domain=
 - c. Right domain=



21. Now, put the domain together with the equations to write the piecewise function for the graph.



Practice: Graph the following (on the same graph) $f(x) = 1 - x, if - 2 \le x \le 1$ $f(x) = x^2, if x > 1$



1. Graph
$$f(x) = \begin{cases} \frac{2x - 1, if \ x \le 1}{3x + 1, if \ x > 1} \end{cases}$$

2. Graph
$$f(x) = \begin{cases} x^2 - 1 & x \le 0 \\ 2x - 1 & 0 < x \le 5 \\ 3 & x > 5 \end{cases}$$

Day 5 Classwork

Part I. Carefully graph each of the following. Identify whether or not he graph is a function. Then, evaluate the graph at any specified domain value. You may use your calculators to help you graph, but you must sketch it carefully on the grid!

1.
$$f(x) = \begin{cases} x+5 & x < -2 \\ x^2+2x+3 & x \ge -2 \end{cases}$$

Function? Yes or No

$$f(-4) =$$

$$f(-2) =$$

2.
$$f(x) = \begin{cases} 2x+1 & x \ge 1 \\ x^2+3 & x < 1 \end{cases}$$

Function? Yes or No

f(-2) =

$$f(1) =$$

3.
$$f(x) = \begin{cases} -2x+1 & x \le 2\\ 5x-4 & x > 2 \end{cases}$$

Function? Yes or No

$$f(-4) =$$

f(8) =

f(2) =



4.
$$f(x) = \begin{cases} x^2 - 1 & x \le 0 \\ 2x - 1 & 0 < x \le 5 \\ 3 & x > 5 \end{cases}$$

Function? Yes or No

$$f(-2) =$$

$$f(0) =$$

5.

$$f(5) =$$

$$f(x) = \begin{cases} x^2 & x \le 0 \\ -x^2 + 4 & x > 0 \end{cases}$$

Function? Yes or No

f(-4) =f(0) =f(3) =

6.
$$f(x) = \begin{cases} 5 & x \leq -3 \\ -2x - 3 & x > -3 \end{cases}$$

Function? Yes or No

$$f(-4) =$$

f(0) =

f(3) =



For each of the following functions

- a) Draw the parent function
- b) Describe each transformation
- c) Sketch a final graph
- d) Label the domain and range of the final graph

1.
$$f(x) = -x^3 - 2$$



2. $f(x) = 3\sqrt{x+1}$





Day 6 Notes

Applications of Piecewise Functions

1. A long distance telephone charges 99 cents for any call up to 20 minutes in length and 7 cents for each additional minute. Use bracket notation to write a formula for the cost, C, of a call as a function of its length time, t, in minutes. Graph the function. How much does it cost to talk for 10 minutes? 25 minutes?

2. Suppose a carpet store sells carpet for \$10 per square yard for the 100 sq yards purchased, and then lowers the price to \$7 per square yard after the first 100 yards have been purchased. Find a function, C = f(x), that gives the cost of purchasing any number of square yards of carpet between 0 and 200 square yards. How much does it cost for 50 square yards? 150 square yards?

3. A company charges \$200 a month to organize a company's payroll for up to 20 employees and an additional \$100 a month for each 20 employees over 20. Find a function, P = f(x), that gives the payroll amount for 100 employees in one month. Graph the function.

Day 6 Classwork

- 1. You are a buyer for a grocery store and you are asked to purchase potatoes for the grocery store. The distributor of potatoes tells you that if you buy up to 50 bushels of potatoes, you will pay \$40 per bushel; and for each bushel you purchase above 50 bushels, you will pay \$30 per bushel.
 - a. How much will your grocery store pay in total if you decide to purchase 40 bushels? 60 bushels? 100 bushels?
 - b. Write a function which has as its input values (x-values) the number of bushels of potatoes purchased and outputs the total amount of money that your grocery store will pay for the potatoes.

- 2. A certain country taxes the first \$20,000 of an individual's income at a rate of 15%, and all income over \$20,000 is taxed at 20%.
 - a. Al makes \$16,000. Betty makes \$36,000. How much is each taxed?
 - b. Write a piecewise function T that specifies the total tax on an income of x dollars.

c. Catina is taxed \$5000. What is her income?

3. A museum charges \$40 for a group of 10 or fewer people. A group of more than 10 people must, in addition to the \$40, pay \$2 per person for the number of people above 10. For example, a group of 12 pays \$44 and a group of 15 pays \$50. The maximum group size is 50.

- a. Find a function, C = f(x), that represents the cost as a function of the number of people going to the museum.
- b. How much would the museum charge for a group of 8? Group with 35 people?

Investigation: Step Functions

A step function is a piecewise constant function. In other words, each piece is a function whose values do not vary and are thus constant. This is sometimes called a staircase function.

1. Sketch the graph of the function below. What values of *x* will make this function true? In other words, where is this function defined, or what is its domain? Why is the range not all real numbers? What do you notice that is different from the first two functions. Graph on your calculator to check your work.



2. Every avid ebayer knows that shipping is an important consideration when listing an item for auction. For infrequent selling, there is not much money to be gained or lost on the transaction, but for the diehard, inaccurate shipping costs can lead to stacked losses over time. Knowing the postal rate scale and what to charge for a given item is paramount. The cost C (in dollars) of sending priority mail, depending on the weight (in ounces) of a package up to five pounds is given by the function below

 $\mathbf{C(x)} = \begin{cases} 13.65, & 0 < x \le 15 \\ 17.00, & 15 < x \le 30 \\ 20.25, & 30 < x \le 55 \\ 23.50, & 55 < x \le 70 \\ 26.25, & 70 < x \le 80 \end{cases}$

Graph the function and identify the domain and range.



3. Given the graph of this step function, find a piecewise constant function that matches the graph.



4. Extention:



- a. Given the graph of this function, write the piecewise function f(x) that matches the graph.
- b. Give the domain and range of the function.
- c. Describe the end behavior of f(x) using infinity notation and/or words.

More Applications of Piecewise

1. An air conditioning salesperson receives a base salary of \$2850 per month plus a commission. The commission is 2% of the sales up to and including \$25,000 for the month and 5% of the sales over \$25,000 for the month.

a. Write a piecewise function that relates the salesperson total monthly income based off or his/her sales for the month.

b. Determine the salesperson's monthly income if his/her sales were \$43,000 for the month.

- 2. *Texting Plans*: Texting from T-mobile costs \$.15 per text with no plan. In addition, they offer three other texting plans, shown to the right, that include a certain number of texts with additional texts over costing \$.15 per text. Decide which plans are linear piecewise functions and which are not piecewise (circle your answer).
 - a. No plan Piecewise? Yes or No
 b. 400 text plan Piecewise? Yes or No
 c. 1000 text plan Piecewise? Yes or No
 d. Unlimited text plan Piecewise? Yes or No
 Unlimited Domestic Messages
 \$9.99 per month
 Unlimited Domestic Messages
 \$14.99 per month

3. Write the function rules for each where x is the number of texts and f(x) is the total monthly cost.

a. No plan b. 400 text plan c. 1000 text plan d. Unlimited text	i. No plan	b. 400 text plan	c. 1000 text plan	d. Unlimited text plan
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4. A parking garage in Manhattan charges in this way: For each hour or part of an hour, the garage charges \$10 per hour, with a daily maximum of \$50 per day.

- a. How much will a customer pay if he/she parks for 2 hours? 3.5 hours? 4 hours?
- b. Write a piecewise function that has as its input the number of hours parked and outputs the total price paid by the customer.

5. 2. The charge for a taxi ride is 1.50 for the first 1/8 of a mile, and 0.25 for each additional 1/8 of a mile (rounded to the nearest 1/8 mile).

- a) Make a table showing the cost of a trip as a function of its length. Your table should start at 0 and go up to one mile in 1/8 mile intervals.
- b) Find a function, C = f(x), that represents the cost of the trip as a function of its length.
- c) What is the cost for a 5/8 mile ride?
- d) How far can you go for \$3.00?

Domestic Text, Instant. Picture &

> Any 400 Domestic Messages

Video

Messaging

Day 8: Functions Review

Find the DOMAIN for problems 1 – 4. Write in interval notation.

1. $f(x) = \frac{x}{x^2 - 9}$ 2. $f(x) = \sqrt{2 - x}$ 3. f(x) = 4x + 3 4. $f(x) = \frac{\sqrt{x + 2}}{x^2 + 2x - 3}$

5. The graph of a function f is known. Then the graph of y = f(x-2) may be found by _____

- 6. The graph of a function is known. Then the graph of y=f(-x) may be obtained by a reflection about the _____-axis.
- 7. True or False:
- _____ a) The graph of y = -f(x) is the reflection about the x-axis of the graph of y = f(x).
- b) To obtain the graph of y = f(x + 2) 3, shift the graph of y = f(x) horizontally to the right 2 units and vertically down 3 units.

8. Find the function that is finally graphed after the following transformations are applied to the graph of $y = \sqrt{x}$.

f(x) = f(x) =

a) 1. Shift up 2 units.	b) 1. Reflect about the x-axis	c) 1. Reflect about the y-axis.
2. Reflect about the x-axis.	2. Shift up 2 units.	2. Vertically stretch by 3.
	3. Shift left 3 units.	3. Shift down 2 units.
		4. Shift right 4 units.

9. USE GRAPH PAPER. Draw the parent graph and then show each transformation.. State the domain and range of the function.

- a) $f(x) = x^3 + 4$ b) $f(x) = (x+4)^2$ c) $f(x) = -\frac{1}{2}|x|$
- d) $f(x) = -2(x-3)^2 1$ e) $f(x) = 2\sqrt{-(x)-1}$

f(x) =

10. State the domain in interval notation. Then graph (on graph paper). Then use the graph to state the range.



11. Find $\frac{f(a+h)-f(a)}{h}$, where $h \neq 0$, for the following two functions. a) f(x) = 2x + 3 b) $f(x) = x^2 - 2$

12. Evaluate the piecewise function for f(-2), f(1), and f(4). $f(x) = \begin{cases} x^2 - 2x, & \text{if } x \le 1 \\ 3x + 1, & \text{if } x > 1 \end{cases}$

13. The domestic postage rate for first class letters weighing 12 oz or less is 33 cents for a letter weighing 1 oz or less and 22 cents for each additional ounce (or part of an ounce). Express the postage P as a function of the weight x of a letter, with $0 < x \le 12$.

14. The cost to attend a play at the theater is \$120 for a group of up to ten students. For each student over ten, the cost is \$12 for each additional student.

a. Write a piecewise function to show the cost to attend the play.

c. How much will it cost for 7 students to attend? For 20 students?

15. Using the graph below, identify the domain, range, intervals of increasing, decreasing and/or constant. Then evaluate at the given values.

