Unit 1 Test Study Guide
(Fundamental Skills)

Name: _____

Date: _____ Per: _____

Topic 1: Exponents and Polynomial Expressions

Simplify each expression. Write all answers with positive exponents and in standard form.

1. $\left(\frac{8a^{-5}b^{4}}{12a^{-6}b^{-2}}\right)^{2}$

$= \frac{64\,a^{-10}b^{8}}{144\,a^{-12}b^{-4}}$

$= \boxed{\frac{4a^{2}b^{12}}{9}}$

2. $\left(\frac{1}{4}x^{3}y^{-2}\right)^{-2}(-4x^{-1}y)^{-3}$

$= \left(16x^{-6}y^{4}\right)\cdot\left(-\frac{1}{64}x^{3}y^{-3}\right)$

$= -\frac{1}{4}x^{-3}y \quad = \boxed{\frac{-y}{4x^{3}}}$

3. $(5k^{2}-7k+9)-(-2k+6k^{2}+11)$

$\boxed{-k^{2}-5k-2}$

4. $-5s^{4}t^{3}(3s^{5}t-7s)+4s^{3}\cdot 2s^{6}t^{4}$

$= -15s^{9}t^{4}+35s^{5}t^{3}+8s^{9}t^{4}$

$= \boxed{-7s^{9}t^{4}+35s^{5}t^{3}}$

5. $(-5g+11)(-5g-11)$

$= 25g^{2}+55g-55g-121$

$= \boxed{25g^{2}-121}$

6. $(2r-5)^{3}$

$= (2r-5)(2r-5)(2r-5)$

$= (2r-5)(4r^{2}-20r+25)$

$= 8r^{3}-40r^{2}+50r-20r^{2}+100r-125$

$= \boxed{8r^{3}-60r^{2}+150r-125}$

Completely factor each expression.

7. $12w^{3}+26w^{2}-10w$

$= 2w(6w^{2}+13w-5)$

$= \boxed{2w(3w-1)(2w+5)}$

8. $64c^{4}-121d^{6}$

$= \boxed{(8c^{2}-11d^{3})(8c^{2}+11d^{3})}$

9. $27r^{3}+216$

$= 27(r^{3}+8)$

$= \boxed{27(r+2)(r^{2}-2r+4)}$

10. $32m-162m^{5}$

$= 2m(16-81m^{4})$

$= 2m(4+9m^{2})(4-9m^{2})$

$= \boxed{2m(4+9m^{2})(2+3m)(2-3m)}$

11. $x^{4}+9x^{2}-112$

$= \boxed{(x^{2}+16)(x^{2}-7)}$

12. $8v^{3}+20v^{2}-18v-45$

$= 4v^{2}(2v+5)-9(2v+5)$

$= (4v^{2}-9)(2v+5)$

$= \boxed{(2v+3)(2v-3)(2v+5)}$

Topic 2: Rational Expressions

Simplify each expression.

13. $\frac{10m^3+20m^2}{15m^3+15m^2-30m}$

$= \frac{10m^2(m+2)}{15m(m^2+m-2)}$

$= \frac{2m(m+2)}{3(m+2)(m-1)} = \boxed{\frac{2m}{3(m-1)}}$

14. $\frac{8a^3+24a^2}{18a+54} \cdot \frac{5a^2+43a+24}{10a^2+6a}$

$= \frac{8a^2(a+3)}{18(a+3)} \cdot \frac{(5a+3)(a+8)}{2a(5a+3)}$

$= \frac{8a^2(a+8)}{36a} = \boxed{\frac{2a(a+8)}{9}}$

15. $\frac{7k^2-36k+5}{3k^2-12k-15} \cdot \frac{42k+6}{1-49k^2}$

$= \frac{(7k-1)(k-5)}{3(k-5)(k+1)} \cdot \frac{6(7k+1)}{(1-7k)(1+7k)}$

$= \frac{-6}{3(k+1)} = \boxed{\frac{-2}{k+1}}$

16. $\frac{\frac{3p+1}{p^2-1} - \frac{1}{(p+1)}}{} \quad (p-1)(p-1)$

$= \frac{3p+1-p+1}{(p+1)(p-1)}$

$= \frac{2p+2}{(p+1)(p-1)} = \frac{2(p+1)}{(p+1)(p-1)} = \boxed{\frac{2}{p-1}}$

17. $\frac{\frac{3x}{(2x-3)} - \frac{3}{(x+6)}}{} \quad (x+6)(2x-3)$

$= \frac{3x(x+6)-3(2x-3)}{(2x-3)(x+6)}$

$= \frac{3x^2+18x-6x+9}{(2x-3)(x+6)} = \boxed{\frac{3x^2+12x+9}{(2x-3)(x+6)}}$

18. $\frac{\frac{1}{2} - \frac{r^2}{18}}{\frac{1}{3} - \frac{1}{r}} \cdot \frac{18}{18} \cdot \frac{r}{r} \cdot \frac{3}{3}$

$= \frac{9-r^2}{18} \cdot \frac{3r}{r-3}$

$= \frac{(3+r)(3-r)}{18} \cdot \frac{3r}{r-3}$

$= \boxed{-\frac{r(3+r)}{6}}$

Topic 3: Radicals and Rational Exponents

Simplify each expression.

19. $\sqrt{294p^7}$

$= \sqrt{49p^6}\sqrt{6p}$

$= \boxed{7p^3\sqrt{6p}}$

20. $2\sqrt[3]{-104x^6y^7}$

$= 2\sqrt[3]{-8x^6y^6}\sqrt[3]{13y}$

$= \boxed{-4x^2y^2\sqrt[3]{13y}}$

21. $-5\sqrt[4]{64m^{11}}$

$= -5\sqrt[4]{16m^8}\sqrt[4]{4m^3}$

$= \boxed{-10m^2\sqrt[4]{4m^3}}$

22. $-4\sqrt{27}+6\sqrt{45}-\sqrt{75}$

$= -4\sqrt{9}\sqrt{3}+6\sqrt{9}\sqrt{5}-\sqrt{25}\sqrt{3}$

$= -12\sqrt{3}+18\sqrt{5}-5\sqrt{3}$

$= \boxed{-17\sqrt{3}+18\sqrt{5}}$

23. $-2\sqrt[3]{2k^5} \cdot \sqrt[3]{-40k^2}$

$= -2\sqrt[3]{-80k^7}$

$= -2\sqrt[3]{8k^6} \cdot \sqrt[3]{10k}$

$= \boxed{4k^2\sqrt[3]{10k}}$

24. $2\sqrt{5}(\sqrt{10}-\sqrt{20})$

$= 2\sqrt{50} - 2\sqrt{100}$

$= 2\sqrt{25}\sqrt{2} - 2 \cdot 10$

$= \boxed{-20 + 10\sqrt{2}}$

25. $(3\sqrt{2}-7)^2$

$= (3\sqrt{2}-7)(3\sqrt{2}-7)$

$= 9\sqrt{4} - 21\sqrt{2} - 21\sqrt{2} + 49$

$= 18 - 42\sqrt{2} + 49$

$= \boxed{67 - 42\sqrt{2}}$

26. $\frac{24\sqrt{112}}{16\sqrt{2}} = \frac{3}{2}\sqrt{56}$

$= \frac{3}{2} \cdot 2\sqrt{14}$

$= \boxed{3\sqrt{14}}$

27. $\frac{(2+\sqrt{3})(5-\sqrt{3})}{(5+\sqrt{3})(5-\sqrt{3})}$

$= \frac{10 - 2\sqrt{3} + 5\sqrt{3} - \sqrt{9}}{25 - 5\sqrt{3} + 5\sqrt{3} - \sqrt{9}}$

$= \boxed{\frac{7 + 3\sqrt{3}}{22}}$

28. Rewrite in exponential form: $\sqrt{2k^5}$

$= \boxed{(2k^5)^{1/2}}$ or $\boxed{2^{\frac{1}{2}} k^{\frac{5}{2}}}$

29. Rewrite in simplest radical form: $(24x^7)^{\frac{1}{3}}$

$= \sqrt[3]{24x^7}$

$= \sqrt[3]{8x^6}\sqrt[3]{3x} = \boxed{2x^2\sqrt[3]{3x}}$

Simplify each expression. Write your answer in simplest radical form.

30. $k^{\frac{1}{4}} \cdot k^{\frac{3}{2}} = k^{7/4}$

$= \sqrt[4]{k^7}$

$= \boxed{k\sqrt[4]{k^3}}$

31. $\frac{u^{\frac{1}{3}} \cdot u^{-\frac{5}{6}}}{u^{-2}} = \frac{u^{-1/2}}{u^{-2}}$

$= u^{3/2}$

$= \sqrt{u^3} = \boxed{u\sqrt{u}}$

Topic 4: Complex Numbers

Simplify each expression.

32. $i^{52} = (i^4)^{13} = 1$

33. $(-3i^7)^3 \cdot 2i^{12}$

$= -27i^{21} \cdot 2i^{12}$

$= -54i^{33} = -54(i^4)^8 \cdot i = \boxed{-54i}$

34. $-i(5-i) + 2(3-7i)$

$= -5i + i^2 + 6 - 14i$

$= -19i - 1 + 6$

$= \boxed{5 - 19i}$

35. $(-2+9i)^2$

$= (-2+9i)(-2+9i)$

$= 4 - 18i - 18i + 81i^2$

$= 4 - 36i - 81 = \boxed{-77 - 36i}$

36. $\frac{(-6-10i)i}{(9i)i} = \frac{-6i - 10i^2}{9i^2}$

$= \frac{-6i + 10}{-9} = \boxed{\frac{-10 + 6i}{9}}$

37. $\frac{(-7-4i)(-8-i)}{(-8+i)(-8-i)} = \frac{56 + 7i + 32i + 4i^2}{64 + 8i - 8i - i^2}$

$= \frac{56 + 39i - 4}{64 + 1}$

$= \frac{52 + 39i}{185} = \boxed{\frac{4+3i}{5}}$

Solve each equation.

38. $8-4(n-1)=-2n+18$

$8-4n+4=-2n+18$

$-4n+12=-2n+18$

$-2n=6$

$\boxed{n=-3}$

39. $\frac{15}{4}\left(12x-\frac{8}{3}\right)=9(2+5x)$

$45x-10=18+45x$

$-10=18$

$\boxed{\emptyset}$

40. $\frac{5p+q^2}{7}=8$ (solve for q)

$5p+q^2=56$

$q^2=56-5p$

$\boxed{q=\sqrt{56-5p}}$

41. $4a-7b=ab+3$ (solve for a)

$4a-ab=7b+3$

$a(4-b)=7b+3$

$\boxed{a=\frac{7b+3}{4-b}}$

Solve each equation. Be sure to check for extraneous solutions.

42. $|5-2x|=11$

$5-2x=11$ $\quad$ $5-2x=-11$

$-2x=6$ $\quad$ $-2x=-16$

$x=-3$ $\quad$ $x=8$

$\boxed{x=\{-3,8\}}$

43. $|4p-9|=6+p$

$4p-9=6+p$ $\quad$ $4p-9=-6-p$

$3p=15$ $\quad$ $5p=3$

$p=5$ $\quad$ $p=\frac{3}{5}$

$\boxed{p=\{\frac{3}{5},5\}}$

44. $-7|10+3m|-7=-63$

$-7|10+3m|=-56$

$|10+3m|=8$

$10+3m=8$ $\quad$ $10+3m=-8$

$3m=-2$ $\quad$ $3m=-18$

$m=-\frac{2}{3}$ $\quad$ $m=-6$

$\boxed{m=\{-6,-\frac{2}{3}\}}$

45. $-5-|-3-4x|=11$

$-|-3-4x|=16$

$|-3-4x|=-16$

$\boxed{\emptyset}$

Topic 6: Quadratic Equations

Solve each equation. Simplify all irrational and complex solutions.

46. $2w^2+3w-5=0$

$(2w+5)(w-1)=0$

$w=\frac{-5}{2}$ | $w=1$

$\boxed{w=\{-\frac{5}{2},1\}}$

47. $r^2+9r+65=-r$

$r^2+10r=-65$

$r^2+10r+25=-65+25$

$(r+5)^2=-40$

$r+5=\pm\sqrt{-40}$

$\boxed{r=\{-5\pm 2i\sqrt{10}\}}$

48. $36c^2+8=4$

$36c^2=-4$

$c^2=-\frac{1}{9}$

$c=\pm\sqrt{-\frac{1}{9}}$

$\boxed{c=\pm\frac{1}{3}i}$

49. $4y^2-28=8y$

$4y^2-8y-28=0$

$y=\frac{8\pm\sqrt{(-8)^2-4(4)(-28)}}{2(4)}$

$y=\frac{8\pm\sqrt{512}}{8}$

$y=\frac{8\pm 16\sqrt{2}}{8}$

$\boxed{y=\{1\pm 2\sqrt{2}\}}$

50. Janell is standing on a set of bleachers and throws a ball into the air at an initial velocity of 35 ft/s. The height of the ball, h, at t seconds is modeled by the equation $h=-16t^2+35t+6$. How many seconds will it take the arrow to reach the ground?

$-16t^2+35t+6=0$

$t=\frac{-35\pm\sqrt{35^2-4(-16)(6)}}{2(-16)}$

$t=\frac{-35\pm\sqrt{1609}}{-32}$

$t=-0.16,\ 2.35$

$\boxed{2.35\text{ sec}}$

Topic 7: Rational Equations

Solve each equation. Be sure to check for extraneous solutions.

51. $\frac{k+6}{4-k}=\frac{2}{k-4}$

$(k+6)(k-4)=2(4-k)$

$k^2+2k-24=8-2k$

$k^2+4k-32=0$

$(k+8)(k-4)=0$

$k=-8$ | $k\neq 4$

$\boxed{k=-8}$

52. $\left[\frac{1}{2k^2}-\frac{3}{k}=\frac{1}{k^2}\right]\cdot 2k^2$

$1-6k=2$

$-6k=1$

$\boxed{k=-\frac{1}{6}}$

53. $u+3=\frac{u^2+3u-4}{u-2}+\frac{3}{u-2}$ $u-2$

$(u+3)(u-2)=u^2+3u-4+3$

$u^2+u-6=u^2+3u-1$

$-6=2u-1$

$-5=2u$

$\frac{-5}{2}=u$

54. $\frac{y+3}{y-5}+\frac{y^2-8y+12}{y^2-4y-5}=\frac{6y-6}{y+1}$ y^2-4y-5

$(y+3)(y+1)+y^2-8y+12=(6y-6)(y-5)$

$y^2+4y+3+y^2-8y+12=6y^2-36y+30$

$2y^2-4y+15=6y^2-36y+30$

$0=4y^2-32y+15$

$0=(2y-15)(2y-1)$

$y=\frac{15}{2}$ | $y=\frac{1}{2}$

$y=\left\{\frac{1}{2},\frac{15}{2}\right\}$

Topic 8: Linear & Absolute Value Inequalities

Solve, graph, and write each solution in interval notation.

55. $-6(4d+3)+8<110$

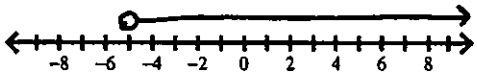
$-24d-18+8<110$

$-24d-10<110$

$-24d<120$

$d>-5$

$(-5,\infty)$



56. $|2x+7|\leq 3$

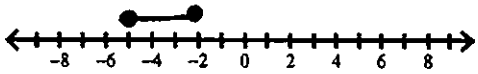
$2x+7\leq 3$ $\quad$ $2x+7\geq -3$

$2x\leq -4$ $\quad$ $2x\geq -10$

$x\leq -2$ $\quad$ $x\geq -5$

$-5\leq x\leq -2$

$[-5,-2]$



57. $-3|-10x-2|<-54$

$|-10x-2|>18$

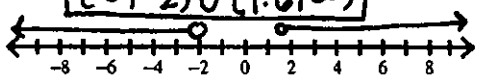
$-10x-2>18$ $\quad$ $-10x-2<-18$

$-10x>20$ $\quad$ $-10x<-16$

$x<-2$ $\quad$ $x>1.6$

$x<-2$ or $x>1.6$

$(-\infty,-2)\cup(1.6,\infty)$



58. $2-3|-3t+6|\geq -61$

$-3|-3t+6|\geq -63$

$|-3t+6|\leq 21$

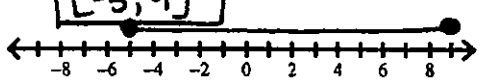
$-3t+6\leq 21$ $\quad$ $-3t+6\geq -21$

$-3t\leq 15$ $\quad$ $-3t\geq -27$

$t\geq -5$ $\quad$ $t\leq 9$

$-5\leq t\leq 9$

$[-5,9]$



Unit 2 Test Study Guide
(Functions & Their Graphs)

Name: _____

Date: _____ Per: _____

Topic 1: Evaluating Functions

For questions 1 and 2, evaluate the following, given $f(x)=\frac{x-2}{2x+3}$.

1. $f(9)$

$$\frac{9-2}{2(9)+3}=\frac{7}{21}=\boxed{\frac{1}{3}}$$

2. $f(x-1)$

$$\frac{x-1-2}{2(x-1)+3}=\boxed{\frac{x-3}{2x+1}}$$

For questions 3 and 4, evaluate the following, given $g(x)=3x-x^2$.

3. $g(2x-1)$

$$3(2x-1)-(2x-1)^2$$
$$=6x-3-(4x^2-4x+1)$$
$$=\boxed{-4x^2+10x-4}$$

4. $g(-3x)$

$$3(-3x)-(-3x)^2$$
$$=-9x-9x^2$$
$$=\boxed{-9x^2-9x}$$

For questions 5 and 6, evaluate the following, given $h(x)=\begin{cases}|-4x+7| & \text{if } x<-3\\ -x^3+2x^2 & \text{if } x\geq -3\end{cases}$.

5. $h(-7)$

$$|-4(-7)+7|$$
$$=|28+7|=\boxed{35}$$

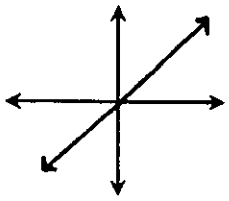
6. $h(-3)$

$$-(-3)^3+2(-3)^2$$
$$=27+18=\boxed{45}$$

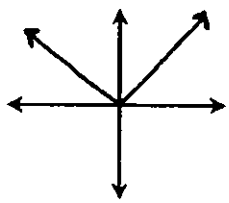
Topic 2: Parent Functions, Transformations, and Graphing

For each function family below, give the parent function and sketch the shape of its graph.

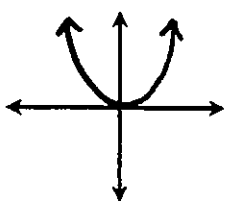
7. Linear $f(x)=x$



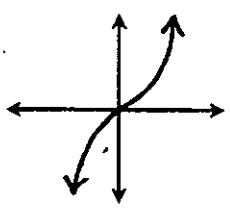
8. Absolute Value $f(x)=|x|$



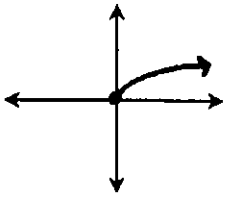
9. Quadratic $f(x)=x^2$



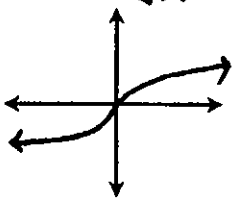
10. Cubic $f(x)=x^3$



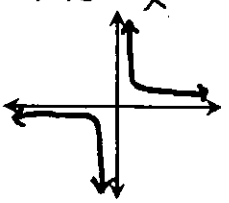
11. Square Root $f(x)=\sqrt{x}$



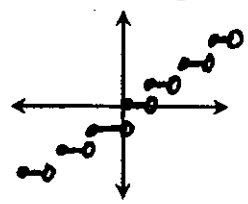
12. Cube Root $f(x)=\sqrt[3]{x}$



13. Reciprocal $f(x)=\frac{1}{x}$



14. Greatest Integer $f(x)=[\![x]\!]$



15. If the quadratic parent function is reflected in the y-axis and vertically compressed by a factor of ½, write an equation to represent the **new function.**

$f(x) = \frac{1}{2}(-x)^2$

16. If the cube root parent function is horizontally stretched by a factor of 4, then translated 5 units right and 3 units up, write an equation to represent the **new function.**

$f(x) = \sqrt[3]{\frac{1}{4}(x-5)} + 3$

17. The absolute value parent function has transformations applied such that it creates an absolute maximum at (-2, 7). Write an eqution that could represent this **new function.**

$f(x) = -|x+2| + 7$

18. A certain function is vertically stretched by a factor of 2, horizontally compressed by ¼, and translated down 6. The new function is represented by $f(x) = 6[\![8x]\!] - 2$. Write the equation of the **original function.**

$f(x) = 3[\![2x]\!] + 4$

19. Describe all transformations from the parent function given the function below.

$f(x) = -3\left(\frac{1}{2}x\right)^3 + 7$

- Vert stretch by 3
- Horiz stretch by 2
- Reflect in x-axis
- Translate up 7

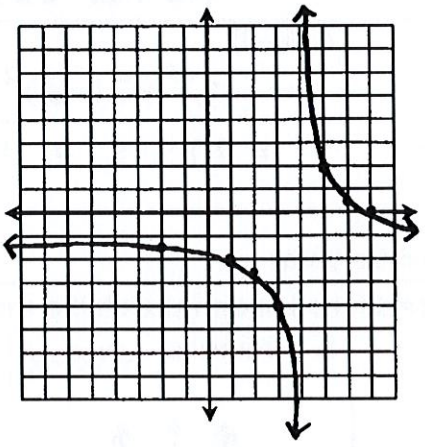
20. Describe all transformations from the parent function given the function below.

$f(x) = \frac{2}{3}\sqrt{-(x-5)} + 2$

- Vert. comp. by 2/3
- Reflect in y-axis
- Translate right 5, up 2

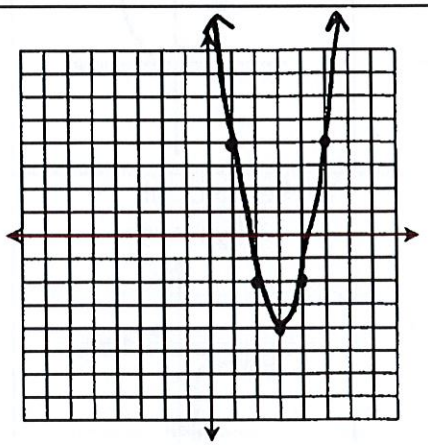
Graph each function and identify all key characteristics.

21. $f(x) = \frac{3}{x-4} - 1$



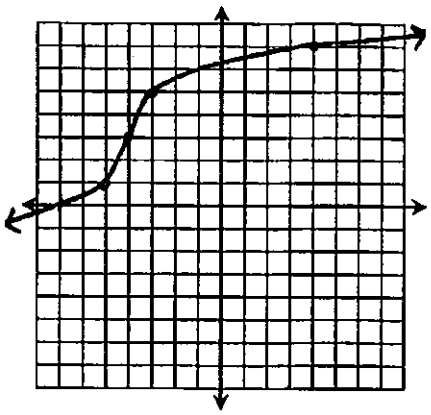
Domain: $\{x \mid x \neq 4\}$	**Range:** $\{y \mid y \neq -1\}$
x-int: (7, 0)	**y-int:** (0, -1.75)
Extrema None	
Increasing Interval: None	
Decreasing Interval: $(-\infty, 4), (4, \infty)$	
End Behavior: As $x \to \infty$, $f(x) \to -1$ As $x \to -\infty$, $f(x) \to -1$	

22. $f(x) = 2(x-3)^2 - 4$



Domain: $\mathbb{R}$	**Range:** $\{y \mid y \geq -4\}$
x-int: (1.6, 0), (4.4, 0)	**y-int:** (0, 14)
Extrema (3, -4) - Abs. Minimum	
Increasing Interval: (3, ∞)	
Decreasing Interval: (-∞, 3)	
End Behavior: As $x \to \infty$, $f(x) \to \infty$ As $x \to -\infty$, $f(x) \to \infty$	

23. $f(x) = 2\sqrt[3]{(x+4)} + 3$

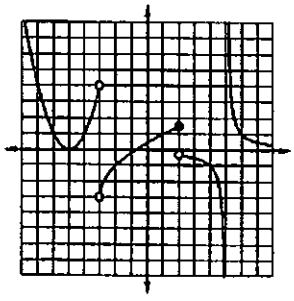


Domain: $\mathbb{R}$	**Range:** $\mathbb{R}$
***x*-int:** $(-7.375, 0)$	***y*-int:** $(0, 6.17)$
Extrema None	
Increasing Interval: $(-\infty, \infty)$	
Decreasing Interval: None	
End Behavior: As $x \to \infty$, $f(x) \to \infty$ As $x \to -\infty$, $f(x) \to -\infty$	

Topic 3: Piecewise Functions

Identify the domain and range of each graph below. State the location and type of any discontinuities.

24.

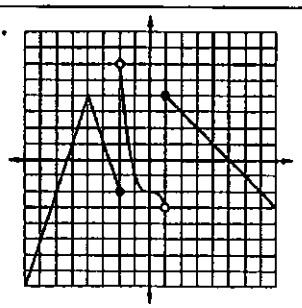


Domain: $\{x \mid x \neq -3, 5\}$

Range: $\mathbb{R}$

Discontinuities:
$x = -3$; jump
$x = 2$; jump
$x = 5$; infinit

25.



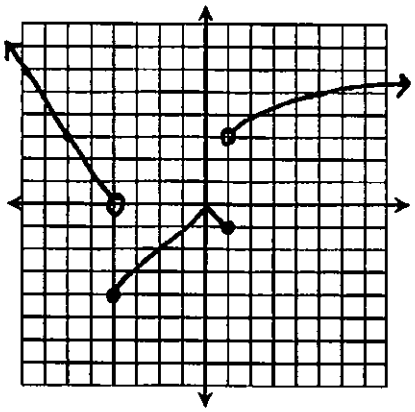
Domain: $\mathbb{R}$

Range: $\{y \mid y < 6\}$

Discontinuities:
$x = -2$; jump
$x = 1$; jump

26. Graph the function below. Identify the domain and range, then, state the location and type of any discontinuities.

$$f(x) = \begin{cases} -\frac{3}{2}x - 6 & \text{if } x < -4 \\ -|x| & \text{if } -4 \le x \le 1 \\ \sqrt{x-1} + 3 & \text{if } x > 1 \end{cases}$$



Domain: $\mathbb{R}$

Range: $\{y \mid y \geq -4\}$

Discontinuities:
$x = -4$; jump
$x = 1$; jump

Topic 4: Average Rate of Change

Find the average rate of change of the function on the given interval.

27. $f(x) = 2x^2 - 3x + 1$; $[-3, 2]$

$$m = \frac{3 - 28}{2 + 3} = \frac{-25}{5} = \boxed{-5}$$

28. $f(x) = \frac{2x-1}{x+3}$; $[-10, -5]$

$$m = \frac{\frac{11}{2} - 3}{-5 + 10} = \frac{\frac{5}{2}}{5} = \boxed{\frac{1}{2}}$$

29. A football is kicked from a point on the ground such that its height $h(t)$, in feet, is given by the equation $h(t) = -16t^2 + 80t$, where t is time in seconds. Find the average rate of change in the height of the ball from when it reaches its maximum height until it reaches the ground.

$t = \frac{-80}{2(-16)} = 2.5$ $\quad [2.5, 5]$

Max: (2.5, 100) $\quad \frac{0-100}{5-2.5} = -\frac{100}{2.5} =$ **-40 ft/sec**

Topic 5: Tests for Symmetry / Even & Odd Functions

Use the graph to determine if the relations given below are symmetrical to the x-axis, y-axis, and/or origin. Confirm your answer algebraically.

30. $x^2 - y^2 = 4$

$(-x)^2 - y^2 = 4$

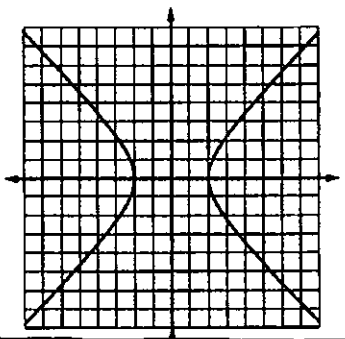
$x^2 - y^2 = 4$ ✓

$x^2 - (-y)^2 = 4$

$x^2 - y^2 = 4$ ✓

$(-x)^2 - (-y)^2 = 4$

$x^2 - y^2 = 4$ ✓



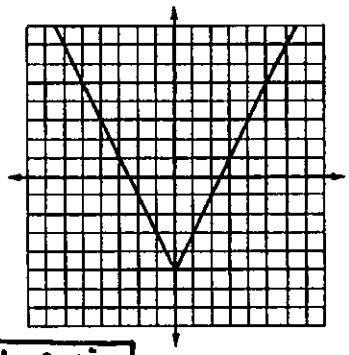
Sym. to x, y, & origin

31. $y = |2x| - 5$

$y = |2(-x)| - 5$

$y = |-2x| - 5$

$y = |2x| - 5$ ✓



Sym. to y-axis

Determine whether the function below is even, odd, or neither. Prove your answer algebraically.

32. $f(x) = -3x^3 + 5x$

$f(-x) = -3(-x)^3 + 5(-x)$

$= 3x^3 - 5x$

Odd (Sym. to origin)

33. $f(x) = 5x^2 + 2x - 1$

$f(-x) = 5(-x)^2 + 2(-x) - 1$

$= 5x^2 - 2x - 1$

Neither

Topic 6: Function Operations & Compositions of Functions

Use $f(x) = 3 - 2x$, $g(x) = \sqrt{x+7}$, and $h(x) = x^2 - 5x$ to find each function below. Be sure to state any domain restrictions, wherever necessary.

34. $(g + f)(x)$

$\sqrt{x+7} + 3 - 2x$

$= \sqrt{x+7} - 2x + 3$

D: $x \geq -7$

35. $(h \cdot f)(x)$

$(x^2 - 5x)(3 - 2x)$

$= 2x^3 + 13x^2 - 15x$

36. $\left(\frac{f}{h}\right)(x)$

$\frac{3-2x}{x^2-5x}$

D: $x \neq 0, 5$

Use $f(x) = -x^2 - 2x$, $g(x) = \sqrt{x+7}$, and $h(x) = 3x - 1$ to find each function below. Give the domain for each.

37. $(h \circ f)(x)$

$3(-x^2-2x)-1$

$= \boxed{-3x^2-6x-1}$

D: $\mathbb{R}$

38. $(f \circ g)(x)$

$-(\sqrt{x+7})^2 - 2(\sqrt{x+7})$

$= -(x+7) - 2\sqrt{x+7}$

$= \boxed{-x-7-2\sqrt{x+7}}$

D: $x \geq -7$

39. $(f \circ h)(x)$

$-(3x-1)^2 - 2(3x-1)$

$= -(9x^2-6x+1) - 6x+2$

$= \boxed{-9x^2+1}$

D: $\mathbb{R}$

Given $h(x)$ below, find two functions, f and g, such that $(f \circ g)(x) = h(x)$.

40. $h(x) = \frac{5}{x-9} - 2$

$f(x) = \frac{5}{x} - 2$

$g(x) = x - 9$

41. $h(x) = -\sqrt{2(x+5)} + 7$

$f(x) = -\sqrt{2x} + 7$

$g(x) = x + 5$

Use $f(x) = |10 - 2x|$, $g(x) = \sqrt[3]{2x-3}$, and $h(x) = \frac{1}{2}x + 5$ to evaluate each function below.

42. $(g - f)(15)$

$g(15) = \sqrt[3]{2(15)-3} = 3$

$f(15) = |10 - 2(15)| = 20$

$3 - 20 = \boxed{-17}$

43. $\left(\frac{h}{g}\right)(-12)$

$h(-12) = \frac{1}{2}(-12) + 5 = -1$

$g(-12) = \sqrt[3]{2(-12)-3} = -3$

$\frac{-1}{-3} = \boxed{\frac{1}{3}}$

44. $(g \circ h)(-6)$

$h(-6) = \frac{1}{2}(-6) + 5$

$= 2$

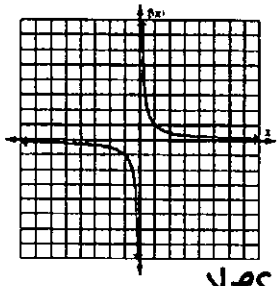
$g(2) = \sqrt[3]{2(2)-3}$

$= \boxed{1}$

Topic 7: Inverse Functions

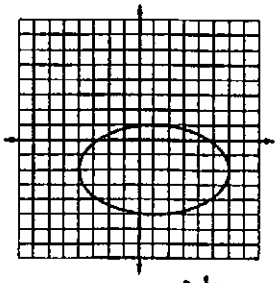
Determine if the graph represents a one-to-one function.

45.



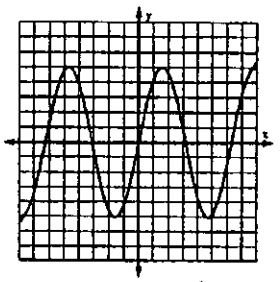
Yes

46.



No

47.



No

Unit 3 Test Study Guide
(Power, Polynomial, and Rational Functions)

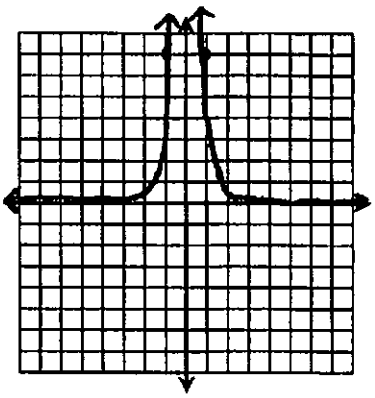
Name: _____

Date: _____ Per: _____

Topic 1: Graphing Power, Polynomial, and Rational Functions

Graph each function and identify all key characteristics.

1. $f(x) = 7x^{-4}$



Domain: $\{x \mid x \neq 0\}$ Range: $\{y \mid y > 0\}$

x-intercept(s): None

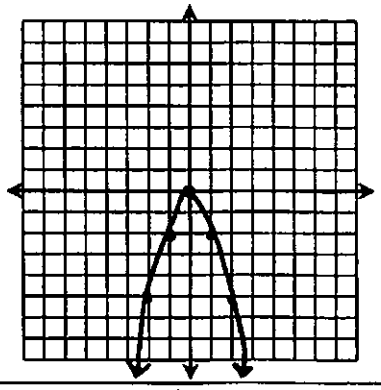
y-intercept: None

Inc. Interval(s): $(-\infty, 0)$

Dec. Interval(s): $(0, \infty)$

End Behavior:
As $x \to \infty$, $f(x) \to 0$
As $x \to -\infty$, $f(x) \to 0$

2. $f(x) = -2x^{\frac{4}{3}}$



Domain: $\mathbb{R}$ Range: $\{y \mid y \leq 0\}$

x-intercept(s): $(0,0)$

y-intercept: $(0,0)$

Inc. Interval(s): $(-\infty, 0)$

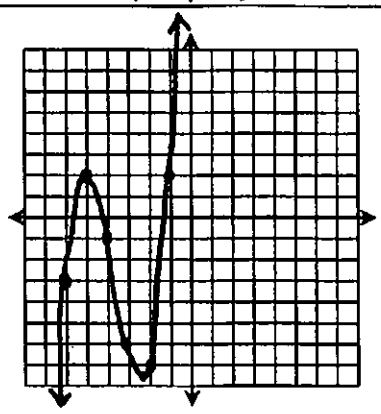
Dec. Interval(s): $(0, \infty)$

End Behavior:
As $x \to \infty$, $f(x) \to -\infty$
As $x \to -\infty$, $f(x) \to -\infty$

3. $f(x) = x^3 + 11x^2 + 35x + 27$

Max $(-5, 2)$

Min $(-2.\overline{3}, -7.48)$



Domain: $\mathbb{R}$ Range: $\mathbb{R}$

x-intercept(s): $(-5.66, 0)$, $(-4.2, 0)$, $(-1.13, 0)$

y-intercept: $(0, 27)$

Inc. Interval(s): $(-\infty, -5)$, $(-2.\overline{3}, \infty)$

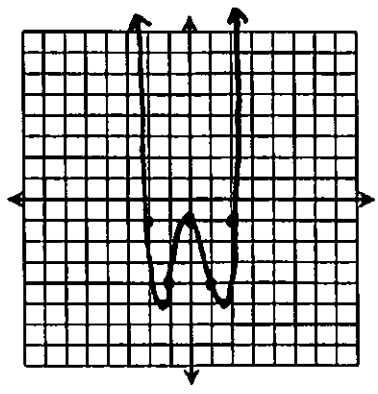
Dec. Interval(s): $(-5, -2.\overline{3})$

End Behavior:
As $x \to \infty$, $f(x) \to \infty$
As $x \to -\infty$, $f(x) \to -\infty$

4. $f(x) = x^4 - 4x^2 - 1$

Min $(-1.41, -5)$
$(1.41, -5)$

Max $(0, -1)$



Domain: $\mathbb{R}$ Range: $\{y \mid y \geq -5\}$

x-intercept(s): $(\pm 2.06, 0)$

y-intercept: $(0, -1)$

Inc. Interval(s): $(-1.41, 0)$, $(1.41, \infty)$

Dec. Interval(s): $(-\infty, -1.41)$, $(0, 1.41)$

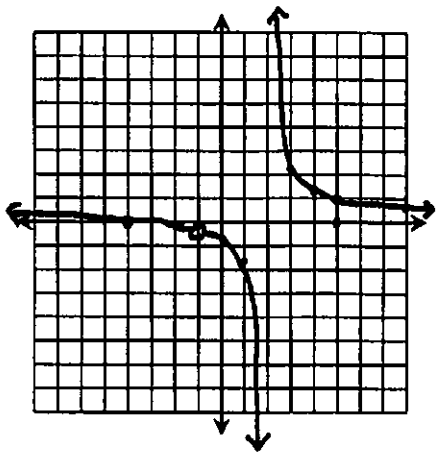
End Behavior:
As $x \to \infty$, $f(x) \to \infty$
As $x \to -\infty$, $f(x) \to \infty$

5. $f(x) = \dfrac{x^2+5x+4}{3x^2-3x-6}$

$= \dfrac{(x+4)(x+1)}{3(x-2)(x+1)}$

$= \dfrac{x+4}{3(x-2)}$

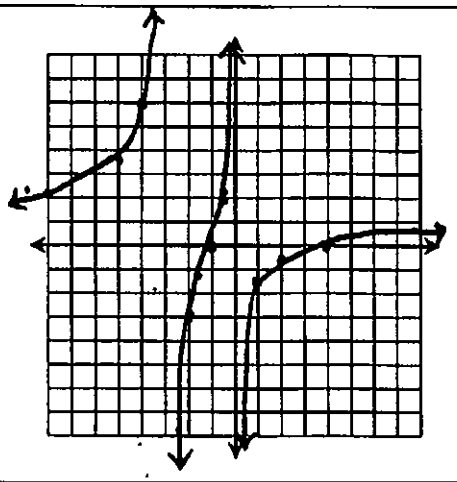
Hole: $x=-1$



Domain: $\{x \mid x \neq -1, 2\}$	Range: $\{y \mid y \neq -\frac{1}{3}, \frac{1}{3}\}$
x-intercept(s): $(-4, 0)$	
y-intercept: $(0, -2/3)$	
Inc. Interval(s): None	
Dec. Interval(s): $(-\infty, 2), (2, \infty)$	
VA: $x=2$	HA: $y=1/3$
SA: None	
Holes: $(-1, -1/3)$	

6. $f(x) = \dfrac{x^2-3x-4}{x^2+3x}$

$= \dfrac{(x-4)(x+1)}{x(x+3)}$



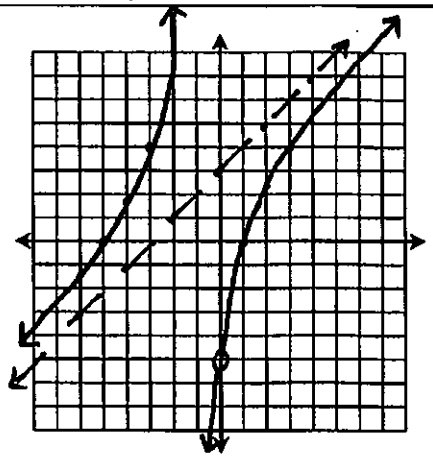
Domain: $\{x \mid x \neq -3, 0\}$	Range: $\{y \mid y \neq 1\}$
x-intercept(s): $(-1, 0), (4, 0)$	
y-intercept: None	
Inc. Interval(s): $(-\infty, -3), (-3, 0), (0, \infty)$	
Dec. Interval(s): None	
VA: $x=-3, x=0$	HA: $y=1$
SA: None	
Holes: None	

7. $f(x) = \dfrac{2x^3+8x^2-10x}{2x^2+2x}$

$= \dfrac{2x(x+5)(x-1)}{2x(x+1)}$

$= \dfrac{x^2+4x-5}{x+1}$

$-1 \mid 1 \quad 4 \quad -5$
$\quad\quad\downarrow \quad -1 \quad -3$
$\quad\quad 1 \quad 3 \quad -8$

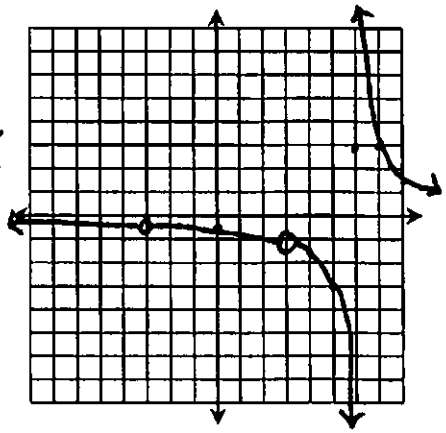


Domain: $\{x \mid x \neq -1, 0\}$	Range: $\mathbb{R}$
x-intercept(s): $(-5, 0), (1, 0)$	
y-intercept: None	
Inc. Interval(s): $(-\infty, -1), (-1, \infty)$	
Dec. Interval(s): None	
VA: $x=-1$	HA: None
SA: $y=x+3$	
Holes: $(0, -5)$	

8. $f(x) = \dfrac{3x^2-27}{x^3-6x^2-9x+54}$

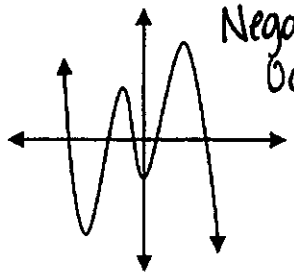
$= \dfrac{3(x+3)(x-3)}{(x-6)(x+3)(x-3)}$

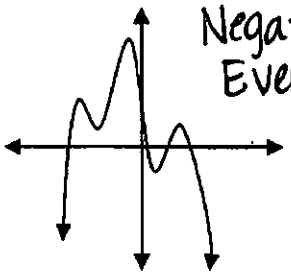
$= \dfrac{3}{x-6}$

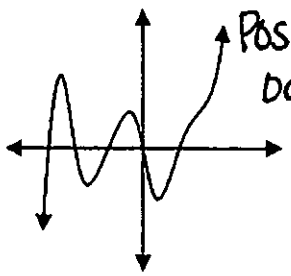


Domain: $\{x \mid x \neq \pm 3, 6\}$	Range: $\{y \mid y \neq -1, -\frac{1}{3}, 0\}$
x-intercept(s): None	
y-intercept: $(0, -0.5)$	
Inc. Interval(s): None	
Dec. Interval(s): $(-\infty, 6), (6, \infty)$	
VA: $x=6$	HA: $y=0$
SA: None	
Holes: $(-3, -1/3), (3, -1)$	

Given the graph of a polynomial functions below, determine the sign of the leading coefficient and whether the function has an even or odd degree.

9.  Negative, odd

10.  Negative, Even

11.  Positive, odd

Topic 2: Dividing Polynomials

Divide the polynomials using synthetic or long division.

12. $(2x^4 - 8x^3 - 22x^2 - 15x + 20) \div (x - 6)$

6	2	-8	-22	-15	20
	↓	12	24	12	-18
	2	4	2	-3	2

$2x^3 + 4x^2 + 2x - 3 + \frac{2}{x-6}$

13. $(5x^5 - 5x^4 + x^3 + 4x - 2) + (x - 1)$

1	5	-5	1	0	4	-2
	↓	5	0	1	1	5
	5	0	1	1	5	-3

$5x^4 + x^2 + x + 5 - \frac{3}{x-1}$

14. $(20x^3 - 4x^2 + 10x + 1) + (5x - 1)$

$$\begin{array}{r} 4x^2 + 0x + 2 \\ 5x-1 \overline{)\,20x^3 - 4x^2 + 10x + 1} \\ -(20x^3 - 4x^2) \\ \hline 0x^2 + 10x \\ -(0x^2 + 0x) \\ \hline 10x + 1 \\ -(10x - 2) \\ \hline 3 \end{array}$$

$4x^2 + 2 + \frac{3}{5x-1}$

15. $(3x^3 + 20x^2 + 14x - 8) \div (x^2 + 6x + 1)$

$$\begin{array}{r} 3x + 2 \\ x^2+6x+1 \overline{)\,3x^3 + 20x^2 + 14x - 8} \\ -(3x^3 + 18x^2 + 3x) \\ \hline 2x^2 + 11x - 8 \\ -(2x^2 + 12x + 2) \\ \hline -x - 10 \end{array}$$

$3x + 2 + \frac{-x-10}{x^2+6x+1}$

Topic 3: The Remainder Theorem

Use the Remainder Theorem to evaluate $f(x)$ at $x = c$.

16. $f(x) = x^3 - 4x^2 + x + 6;\ c = 5$

5	1	-4	1	6
	↓	5	5	30
	1	1	6	36

36

17. $f(x) = 4x^4 - 3x^3 + 7x - 11;\ c = -2$

-2	4	-3	0	7	-11
	↓	-8	22	-44	74
	4	-11	22	-37	63

63

18. $f(x) = x^4 - 6x^3 - 3x^2 + 2x + 2;\ c = 1$

1	1	-6	-3	2	2
	↓	1	-5	-8	-6
	1	-5	-8	-6	-4

-4

19. $f(x) = 2x^4 + 5x^3 + 3x + 4;\ c = -3$

-3	2	5	0	3	4
	↓	-6	3	-9	18
	2	-1	3	-6	22

22

Topic 4: The Factor Theorem

Use the factor theorem to determin which binomials are factors of the functions below.

20. $f(x) = 2x^3 - 3x^2 - 17x - 12$

$(x+1)$:

-1	2	-3	-17	-12
	↓	-2	5	12
	2	-5	-12	0

$(x-4)$:

4	2	-5	-12
	↓	8	12
	2	3	0

$(x+3)$:

-3	2	3
	↓	-6
	2	~~-3~~

- ☑ $(x+1)$
- ☑ $(x-4)$
- ☐ $(x+3)$

21. $f(x) = x^4 + 4x^3 - 8x^2 - 35x - 12$

$(x+2)$:

-2	1	4	-8	-35	-12
	↓	-2	-4	24	22
	1	2	-12	-11	~~10~~

$(x+4)$:

-4	1	4	-8	-35	-12
	↓	-4	0	32	12
	1	0	-8	-3	0

$(x-6)$:

6	1	0	-8	-3
	↓	6	36	168
	1	6	28	~~165~~

- ☐ $(x+2)$
- ☑ $(x+4)$
- ☐ $(x-6)$

Topic 5: Rational Zero Theorem

Use the Rational Zero Theorem to list all possible rational zeros.

22. $f(x) = x^4 + 12x^3 + 7x^2 - 42$

$\pm 1, \pm 2, \pm 3, \pm 6, \pm 7, \pm 14, \pm 21, \pm 42$

23. $f(x) = x^3 - x^2 - 8x^2 + 15$

$\pm 1, \pm 3, \pm 5, \pm 15$

24. $f(x) = 3x^5 - 11x^3 - 15x + 24$

$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{8}{3}$

25. $f(x) = 2x^3 + 5x^2 - 2x + 28$

$\pm 1, \pm 2, \pm 4, \pm 7, \pm 14, \pm 28, \pm \frac{1}{2}, \pm \frac{7}{2}$

Topic 6: Descartes' Rule of Signs

Use Descartes' Rule of Signs to give the possible number of positive and negative real zeros.

26. $f(x) = -x^5 + 18x^3 + 6x^2 - 5x - 4$

$f(-x) = x^5 - 18x + 6x^2 + 5x - 4$

Pos: 2 or 0 Neg: 3 or 1

27. $f(x) = x^4 + 7x^3 - 9x^2 + x - 2$

$f(-x) = x^4 - 7x^3 - 9x^2 - x - 2$

Pos: 3 or 1 Neg: 1

28. $f(x) = 5x^4 - 14x^2 + 9$

$f(-x) = 5x^4 - 14x^2 + 9$

Pos: 2 or 0 Neg: 2 or 0

29. $f(x) = 6x^5 - 3x^4 + 56x^3 - 28x^2 + 64x - 32$

$f(-x) = -6x^5 - 3x^4 - 56x^3 - 28x^2 - 64x - 32$

Pos: 5, 3, or 1 Neg: 0

Topic 7: Zeros of Polynomial Functions & Complete Factorization

Find all zeros and give the complete factorization of the function. Use the Rational Zero Theorem and division when necessary. Simplify all irrational and complex solutions.

30. $f(x) = x^4 + 14x^2 - 72$

$f(x) = (x^2-4)(x^2+18)$

$x = \pm 2$ | $x^2 = -18$

$x = \pm i\sqrt{18}$

$x = \pm 3i\sqrt{2}$

Zeros: $x = \{\pm 2, \pm 3i\sqrt{2}\}$

$f(x) = (x+2)(x-2)(x+3i\sqrt{2})(x-3i\sqrt{2})$

31. $f(x) = 5x^3 + 12x^2 + x - 6$ $\quad \pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{5}, \pm \frac{2}{5}, \pm \frac{3}{5}, \pm \frac{6}{5}$

-1	5	12	1	-6
	↓	-5	-7	6
	5	7	-6	0

$f(x) = (x+1)(5x^2+7x-6)$

$f(x) = (x+1)(5x-3)(x+2)$

Zeros: $x = \{-2, -1, \frac{3}{5}\}$

32. $f(x) = x^3 - 3x^2 - 13x + 15$ $\quad \pm 1, \pm 3, \pm 5, \pm 15$

1	1	-3	-13	15
	↓	1	-2	-15
	1	-2	-15	0

$f(x) = (x-1)(x^2-2x-15)$

$f(x) = (x-1)(x-5)(x+3)$

Zeros: $x = \{-3, 1, 5\}$

33. $f(x) = x^3 + 2x^2 - 17x - 36$ $\quad \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 9, \pm 12, \pm 18, \pm 36$

-4	1	2	-17	-36
	↓	-4	8	36
	1	-2	-9	0

$f(x) = (x+4)(x^2-2x-9)$

$x = -4$ | $x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(9)}}{2(1)}$

$x = \frac{2 \pm \sqrt{40}}{2}$

$x = \frac{2 \pm 2\sqrt{10}}{2}$

$x = 1 \pm \sqrt{10}$

$f(x) = (x+4)(x-(1+\sqrt{10}))(x-(1-\sqrt{10}))$

Zeros: $x = \{-4, 1 \pm \sqrt{10}\}$

Topic 8: Multiplicity

Identify the zeros, their multiplicities, and describe the effect on the graph.

34. $f(x) = x^3(2x-3)^2(x+7)^5$

Zero	Multiplicity	Effect
-7	5	intersects
0	3	intersects
3/2	2	tangent

35. $f(x) = x^6 - 2x^5 - 4x^4 + 8x^3$

$x^5(x-2) - 4x^3(x-2)$

$(x^5 - 4x^3)(x-2)$

$x^3(x^2-4)(x-2)$

$x^3(x+2)(x-2)^2$

Zero	Multiplicity	Effect
-2	1	intersects
0	3	intersects
2	2	tangent

Topic 9: Writing Polynomial Functions Given Zeros

Use the given zeros to write a polynomial function.

36. 0, −1 (multiplicity 2), $\frac{4}{3}$

$x(x+1)^2(3x-4)$

$=(3x^2-4x)(x^2+2x+1)$

$=3x^4+6x^3+3x^2-4x^3-8x^2-4x$

$\boxed{f(x)=3x^4+2x^3-5x^2-4x}$

37. $\pm 5, \pm 3\sqrt{2}$

$(x+5)(x-5)(x+3\sqrt{2})(x-3\sqrt{2})$

$=(x^2-25)(x^2-18)$

$=x^4-18x^2-25x^2+450$

$\boxed{f(x)=x^4-43x^2+450}$

38. −3, −1 − 3*i*

$(x+3)(x-(-1-3i))(x-(-1+3i))$

$=(x+3)(x^2-x(-1+3i)-x(-1-3i)+(-1-3i)(-1+3i))$

$=(x+3)(x^2+x-3ix+x+3ix+10)$

$=(x+3)(x^2+2x+10)$

$=x^3+2x^2+10x+3x^2+6x+30$

$\boxed{f(x)=x^3+5x^2+16x+30}$

Topic 10: Polynomial & Rational Inequalities

Solve the inequality. Use the number line to test intervals.

39. $2x^3+5x^2-32x-80>0$

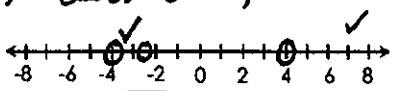
$(x^2-16)(2x+5)>0$ Zeros: $-4, -\frac{5}{2}, 4$

$-5: 2(-5)^3+5(-5)^2-32(-5)-80>0;\ -45>0$ ✗

$-3: 2(-3)^3+5(-3)^2-32(-3)-80>0;\ 7>0$ ✓

$0: 2(0)^3+5(0)^2-32(0)-80>0;\ -80>0$ ✗

$5: 2(5)^3+5(5)^2-32(5)-80>0;\ 135>0$ ✓



$\boxed{(-4,-\frac{5}{2})\cup(4,\infty)}$

40. $x^3-37x+45\le 4x^2+5$

$x^3-4x^2-37x+40\le 0$

$\pm 1, \pm 2, \pm 4, \pm 5, \pm 8, \pm 10, \pm 20, \pm 40$

1	1	−4	−37	40
	↓	1	−3	−40
	1	−3	−40	0

$f(x)=(x-1)(x^2-3x-40)$

$(x-1)(x-8)(x+5)$

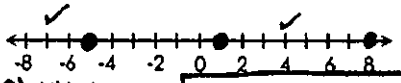
Zeros: $-5, 1, 8$

$-6: (-6)^3-4(-6)^2-37(-6)+40\le 0;\ -98\le 0$ ✓

$0: 0^3-4(0)^2-37(0)+40\le 0;\ 40\le 0$ ✗

$2: 2^3-4(2)^2-37(2)+40\le 0;\ -42\le 0$ ✓

$9: 9^3-4(9)^2-37(9)+40\le 0;\ 112\le 0$ ✗



$\boxed{(-\infty,-5]\cup[1,8]}$

41. $\frac{x^2+x-12}{x-1}>0$

Zeros: $x=-4, 3$

Asym: $x=1$

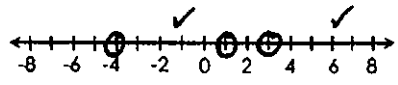
$\frac{(x+4)(x-3)}{x-1}>0$

$-5: \frac{8}{-6}>0;\ -\frac{4}{3}>0$ ✗

$0: \frac{-12}{-1}>0;\ 12>0$ ✓

$2: \frac{-6}{1}>0;\ -6>0$ ✗

$4: \frac{8}{3}>0$ ✓



$\boxed{(-4,1)\cup(3,\infty)}$

42. $\frac{7}{x+3}\le\frac{6}{x+2}$

$\frac{7(x+2)-6(x+3)}{(x+3)(x+2)}\le 0$

Zero: $x=4$

Asym: $x=-3, -2$

$\frac{x-4}{(x+3)(x+2)}\le 0$

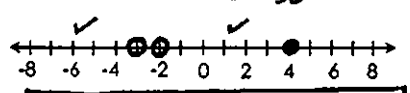
$-4: \frac{-8}{2}\le 0;\ -4\le 0$ ✓

$0: \frac{-4}{6}\le 0;\ -\frac{1}{3}\le 0$ ✓

$-25: \frac{-6.5}{-.25}\le 0;$

$26\le 0$ ✗

$5: \frac{1}{56}\le 0$ ✗



$\boxed{(-\infty,-3)\cup(-2,4]}$