

* $f^{-1}(x), y^{-1}, g(x)$

Inverse Functions Study Guide

all mean inverse of $f(x)$

Name: Control

1. Determine whether the functions $f(x)$ and $g(x)$ are inverses. Justify your results.

a. $f(x) = 3x - 5$ and $g(x) = \frac{x+5}{3}$

$y^{-1} = g(x) = \frac{x+5}{3}$ ✓

$f(x) = 3x - 5$

$y = 3x - 5$

$y + 5 = 3x$

$\frac{y+5}{3} = x \Rightarrow \boxed{y^{-1} = \frac{x+5}{3}}$

b. $f(x) = \frac{1}{x}$ and $g(x) = \frac{1}{x}$

$f(x) = \frac{1}{x}$

$y = \frac{1}{x}$

$xy = 1$

$x = \frac{1}{y}$
 $\boxed{f^{-1}(x) = \frac{1}{x}}$

$y^{-1} = g(x) = \frac{1}{x}$ ✓

c. $f(x) = 13 + x$ and $g(x) = x - 13$

$f(x) = 13 + x$

$y = 13 + x$

$y - 13 = x$

$\boxed{f^{-1}(x) = x - 13}$

$\boxed{y^{-1} = y - 13}$

$y^{-1} = g(x) = x - 13$ ✓

2. Find the inverse function, $g(x)$, of the given function, $f(x)$. Verify your answer using function composition.

a. $f(x) = \frac{\sqrt[3]{x+4}}{2}$

$y = \frac{\sqrt[3]{x+4}}{2} \Rightarrow 2y = \sqrt[3]{x+4}$

$f(g(x)) = \frac{\sqrt[3]{(2x-4)+4}}{2}$

$= \frac{(2x-4)+4}{2} = \frac{2x}{2} = \boxed{x}$ ✓

$\Rightarrow 2y - 4 = \sqrt[3]{x+4} \Rightarrow (2y-4)^3 = x+4$

$\Rightarrow f^{-1}(x) = g(x) = (2x-4)^3$

b. $f(x) = \frac{x}{10} + 5$

$y = \frac{x}{10} + 5 \Rightarrow y - 5 = \frac{x}{10} \Rightarrow 10y - 50 = x$

$f(g(x)) = \frac{(10x-50)}{10} + 5$

$= x - 5 + 5$

$y = \boxed{x}$ ✓

$\Rightarrow f^{-1}(x) = y^{-1} = 10x - 50 = g(x)$

c. $f(x) = \frac{4x-5}{3}$

$y = \frac{4x-5}{3} \Rightarrow 3y = 4x-5 \Rightarrow 3y+5 = 4x$

$f(g(x)) = \frac{4[\frac{3x+5}{4}] - 5}{3} = \frac{3x+5-5}{3}$

$\frac{3y+5}{4} = x \Rightarrow y^{-1} = \frac{3x+5}{4}$

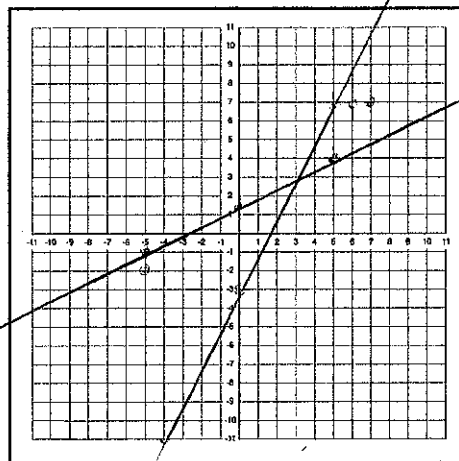
$y = \boxed{x}$ ✓

3. Determine whether the functions $f(x)$ and $g(x)$ are inverses by graphing them:

a. $f(x) = 2x - 3$ and $g(x) = \frac{x+3}{2}$

x	f(x)
0	-3
5	+7
10	+17
-5	-13
-10	-23

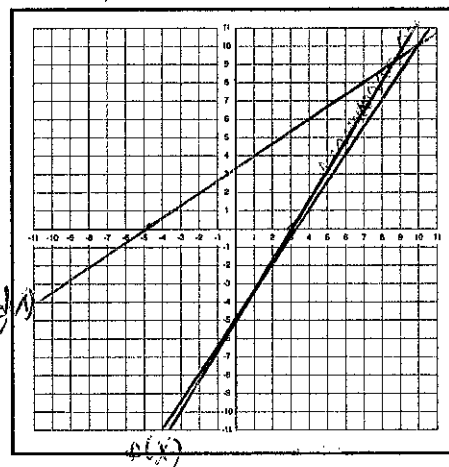
x	f(x)
-5	-1
0	3/2
5	4



b. $f(x) = \frac{3x-10}{2}$ and $g(x) = \frac{2x+10}{3}$

x	f(x)
0	-5
4	1
8	7
10	10
-4	-11

x	g(x)
-5	0
1	4
7	8
10	10
-11	-4



4. The Celsius temperature scale is based on the properties of water, so that the temperature at which water freezes is 0 C and the temperature at which water boils is 100 C. To convert from degrees Celsius to degrees Fahrenheit, you can use the function $F = \frac{9}{5}C + 32$. Find the inverse of this function, which you could use to convert from degrees Fahrenheit to degrees Celsius.

Celsius \rightarrow Fahrenheit | Fahrenheit \rightarrow Celsius

$$F = \frac{9}{5}C + 32$$

$$C = \frac{5F - 160}{9}$$

$$F = \frac{9}{5}C + 32$$

$$F = \frac{9C + 160}{5}$$

$$5F - 160 = 9C$$

5. Find the inverse, $g(x)$, of the given function, $f(x)$.

a. $f(x) = (x+7)^3$

$$y = (x+7)^3$$

$$\sqrt[3]{y} = x+7$$

$$\sqrt[3]{y} - 7 = x$$

$$\Rightarrow \boxed{y^{-1} = \sqrt[3]{x} - 7}$$

b. $f(x) = x^2 + 3, x \geq 0$

$$y = x^2 + 3$$

$$y - 3 = x^2$$

$$\boxed{x = \sqrt{y-3}, x \geq 0}$$