

Cholesterol Activity: Completely Randomized Design vs. Matched Pairs Design

The Purpose: The purpose of this activity is to illustrate how to assign treatments in a completely randomized design, how to assign treatments in a matched pairs design, and how a matched pairs design can improve an experiment.

The Context: A pharmaceutical company has developed a new drug to lower cholesterol and wants to compare it to the drug that is currently being recommended by most doctors. To do this, the company recruits 20 male volunteers, all similar in age. Half of the subjects will be assigned the new drug and the other half will be assigned the current drug.

The Procedure, Part 1: A Completely Randomized Design

1. Give each student one of the 20 envelopes (labeled “Subject 1: initial cholesterol = 299” to “Subject 20: initial cholesterol = 205”). *See the note about modifications below if you have fewer than 20 students or more than 20 students.*
2. Assign each subject a treatment. To do this, use the 10 cards labeled “New” and the 10 cards labeled “Current”. Shuffle the cards and distribute them at random to each subject. To make this more visual, have all the students with the new treatment go to one side of the room and all the students with the current treatment go to the other side of the room.
3. Have students open the envelopes and use the cards inside (plus the treatment cards) to calculate the reduction in cholesterol for their subject. Ask them to not look at the cards in other envelopes or discuss the values on the cards with other students.

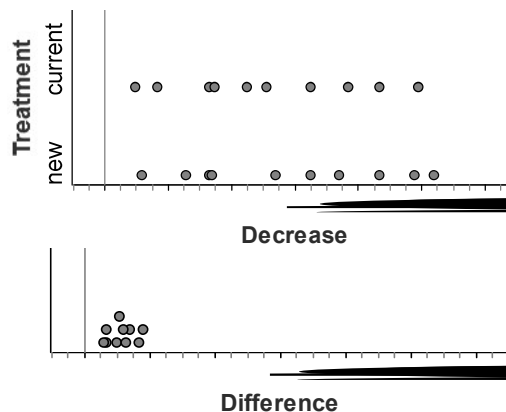
In each envelope there should be 3 items:

- A card with Individual Effect (e.g. “Subject 1: Individual Effect = 69”). This measures the average decrease this individual would have using either treatment.
 - A card with Random Error: “Random Error = RandNorm(0,2)”. This command generates a random number from a normal distribution with a mean of 0 and a SD of 2. The card also instructs students to round this number to the nearest integer. This adds variability due to other factors, such as the office party on Friday or sleeping through breakfast in the morning.
 - A slip of paper with the following formula:
Total Decrease = Individual Effect + Treatment Effect + Random Error
Total Decrease = _____ + _____ + _____ = _____
 - Note: The Treatment Cards that were randomly assigned have the treatment listed (New or Current) as well as the effect of the treatment (+5 or –5). This measures how much effect the treatment will have, *in addition to* the individual effect.
4. Collect data by having each subject report his treatment (New or Current) and his total reduction in cholesterol.
 5. Make a graphical display to compare the reductions for the two treatments. There should be quite a lot of overlap, making it difficult to tell which treatment is better.
 6. Collect the treatment cards and have the students return the other cards to their envelopes.

The Procedure, Part 2: A Matched Pairs Design

1. Ask students to think of variables we could use for pairing the subjects (i.e. forming blocks of size 2). The best variables to use are the ones with the strongest association with the response variable. For example, it is certainly possible that the men with the highest initial cholesterol will have a different response than the men with the lowest initial cholesterol. Furthermore, it would be unfortunate if most of the men receiving one treatment had higher initial cholesterol than most of the men in the other treatment group, which could happen by chance in a completely randomized experiment.
2. Pair the subjects by their initial cholesterol level given on the outside of the envelope. The easiest way to do this is to have all the subjects form a single file line, sorted by their initial cholesterol levels.
3. Have the first pair move forward. Remind students that this pair is a block of size 2. Now, take one “Current” card and one “New” card, shuffle them and give them out at random to the two members of the pair. Have them sit down together so they remember who they are paired with.
4. Do the same thing with the remaining 9 pairs.
5. Have students calculate the reduction in cholesterol for their subjects.
6. Have each pair stand up and report the difference in their reductions (New – Current). For example “In our pair, the subject with New did 8 better than the subject with Current.”
7. Make a graph of the 10 differences (they should all be greater than 0, making it much more obvious that the new treatment is better than the currently used treatment).
8. Ask students to discuss why pairing made the difference in treatments much more obvious. The statistical answer is that we greatly reduced a big source of variability in the results: the individual effect. Overall, there are three sources of variability in the response variable (reduction in cholesterol): the individual effect, the treatment effect, and random error. Since the individual effects are much more varied than the other sources of variability and highly correlated with the initial cholesterol levels, pairing two subjects that have similar individual effects and looking at the difference in their response essentially subtracts out the individual effect from the equation. This leaves only two sources of variability (the treatment effect and random error), making it easier to see the difference in the treatments.

Examples of Graphs:



Modifications: If you have fewer than 20 students, you can use fewer envelopes. Just make sure that you are removing envelopes that would be paired together (similar initial cholesterol). The table on the next page is sorted by initial cholesterol and pairs are identified to make this easier. Don't just remove pairs from one end of the list, however. This will make the decrease in variability less obvious. Instead, remove pairs from the middle, such as pairs 3, 5, and 8.

Also, if you have more than 20 students, then you can add additional envelopes. The second table provides an additional 20 subjects. Again, this table is sorted by initial cholesterol and pairs are identified so you know which pairs to include.

Materials: The cards needed for the envelopes can be found on the following pages. You will need to make extra copies of the formula cards so there are 2 for each envelope. It is a good idea to print each type of card on a different color of paper. The 20 envelopes should be labeled with Subject Number and Initial Cholesterol as in the data table below.

Note: This activity was inspired by "Blocking Activity with Dogs" by Landy Godbold, Dan Teague, Floyd Bullard, and Chris Olsen. http://courses.ncssm.edu/math/Stat_Inst/Stats2007/blocking_activity_with_dogs%202.pdf

Data Table:

| Subject | Initial Cholesterol | Individual Effect |
|---------|---------------------|-------------------|
| 1 | 299 | 69 |
| 2 | 183 | 9 |
| 3 | 191 | 19 |
| 4 | 318 | 80 |
| 5 | 329 | 90 |
| 6 | 197 | 20 |
| 7 | 228 | 39 |
| 8 | 270 | 59 |
| 9 | 310 | 79 |
| 10 | 338 | 99 |
| 11 | 268 | 50 |
| 12 | 229 | 40 |
| 13 | 209 | 30 |
| 14 | 285 | 60 |
| 15 | 303 | 70 |
| 16 | 321 | 89 |
| 17 | 187 | 10 |
| 18 | 350 | 100 |
| 19 | 252 | 49 |
| 20 | 205 | 29 |

Additional Subjects (if needed):

| Subject | Initial Cholesterol | Individual Effect | Pair |
|----------------|----------------------------|--------------------------|-------------|
| 22 | 188 | 12 | 11 |
| 37 | 189 | 13 | 11 |
| 23 | 198 | 22 | 12 |
| 26 | 201 | 23 | 12 |
| 40 | 215 | 32 | 13 |
| 33 | 219 | 33 | 13 |
| 27 | 238 | 42 | 14 |
| 32 | 239 | 43 | 14 |
| 39 | 269 | 52 | 15 |
| 31 | 269 | 53 | 15 |
| 28 | 288 | 62 | 16 |
| 34 | 293 | 63 | 16 |
| 21 | 305 | 72 | 17 |
| 35 | 308 | 73 | 17 |
| 29 | 319 | 82 | 18 |
| 24 | 320 | 83 | 18 |
| 36 | 331 | 92 | 19 |
| 25 | 333 | 93 | 19 |
| 30 | 358 | 102 | 20 |
| 38 | 370 | 103 | 20 |

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|---------------------------------------|---|---------------------------------------|---------------------------------------|
| Subject 1: Individual Effect = 69 | Subject 2: Individual Effect = 9 | Subject 3: Individual Effect = 19 | Subject 4: Individual Effect = 80 |
| Subject 5: Individual Effect = 90 | Subject 6: Individual Effect = 20 | Subject 7: Individual Effect = 39 | Subject 8: Individual Effect = 59 |
| Subject 9: Individual Effect = 79 | Subject 10: Individual Effect = 99 | Subject 11: Individual Effect = 50 | Subject 12: Individual Effect = 40 |
| Subject 13: Individual Effect = 30 | Subject 14: Individual Effect = 60 | Subject 15: Individual Effect = 70 | Subject 16: Individual Effect = 89 |
| Subject 17: Individual Effect = 10 | Subject 18: Individual Effect = 100 | Subject 19: Individual Effect = 49 | Subject 20: Individual Effect = 29 |

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|---|---|---|---|
| <p>Random Error = RandNorm(0,2)</p> <p>(Round to nearest integer)</p> | <p>Random Error = RandNorm(0,2)</p> <p>(Round to nearest integer)</p> | <p>Random Error = RandNorm(0,2)</p> <p>(Round to nearest integer)</p> | <p>Random Error = RandNorm(0,2)</p> <p>(Round to nearest integer)</p> |
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| <p>Random Error = RandNorm(0,2)</p> <p>(Round to nearest integer)</p> | <p>Random Error = RandNorm(0,2)</p> <p>(Round to nearest integer)</p> | <p>Random Error = RandNorm(0,2)</p> <p>(Round to nearest integer)</p> | <p>Random Error = RandNorm(0,2)</p> <p>(Round to nearest integer)</p> |
| <p>Random Error = RandNorm(0,2)</p> <p>(Round to nearest integer)</p> | <p>Random Error = RandNorm(0,2)</p> <p>(Round to nearest integer)</p> | <p>Random Error = RandNorm(0,2)</p> <p>(Round to nearest integer)</p> | <p>Random Error = RandNorm(0,2)</p> <p>(Round to nearest integer)</p> |

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|--|--|
| <p>Total Decrease:</p> $= \frac{\quad}{(\text{indiv. effect})} + \frac{\quad}{(\text{trt. effect})} + \frac{\quad}{(\text{random error})}$ $= \underline{\quad}$ | <p>Total Decrease:</p> $= \frac{\quad}{(\text{indiv. effect})} + \frac{\quad}{(\text{trt. effect})} + \frac{\quad}{(\text{random error})}$ $= \underline{\quad}$ |
| <p>Total Decrease:</p> $= \frac{\quad}{(\text{indiv. effect})} + \frac{\quad}{(\text{trt. effect})} + \frac{\quad}{(\text{random error})}$ $= \underline{\quad}$ | <p>Total Decrease:</p> $= \frac{\quad}{(\text{indiv. effect})} + \frac{\quad}{(\text{trt. effect})} + \frac{\quad}{(\text{random error})}$ $= \underline{\quad}$ |
| <p>Total Decrease:</p> $= \frac{\quad}{(\text{indiv. effect})} + \frac{\quad}{(\text{trt. effect})} + \frac{\quad}{(\text{random error})}$ $= \underline{\quad}$ | <p>Total Decrease:</p> $= \frac{\quad}{(\text{indiv. effect})} + \frac{\quad}{(\text{trt. effect})} + \frac{\quad}{(\text{random error})}$ $= \underline{\quad}$ |
| <p>Total Decrease:</p> $= \frac{\quad}{(\text{indiv. effect})} + \frac{\quad}{(\text{trt. effect})} + \frac{\quad}{(\text{random error})}$ $= \underline{\quad}$ | <p>Total Decrease:</p> $= \frac{\quad}{(\text{indiv. effect})} + \frac{\quad}{(\text{trt. effect})} + \frac{\quad}{(\text{random error})}$ $= \underline{\quad}$ |
| <p>Total Decrease:</p> $= \frac{\quad}{(\text{indiv. effect})} + \frac{\quad}{(\text{trt. effect})} + \frac{\quad}{(\text{random error})}$ $= \underline{\quad}$ | <p>Total Decrease:</p> $= \frac{\quad}{(\text{indiv. effect})} + \frac{\quad}{(\text{trt. effect})} + \frac{\quad}{(\text{random error})}$ $= \underline{\quad}$ |
| <p>Total Decrease:</p> $= \frac{\quad}{(\text{indiv. effect})} + \frac{\quad}{(\text{trt. effect})} + \frac{\quad}{(\text{random error})}$ $= \underline{\quad}$ | <p>Total Decrease:</p> $= \frac{\quad}{(\text{indiv. effect})} + \frac{\quad}{(\text{trt. effect})} + \frac{\quad}{(\text{random error})}$ $= \underline{\quad}$ |

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|---|---|---|---|
| Treatment: New Treatment Effect = +5 | Treatment: New Treatment Effect = +5 | Treatment: New Treatment Effect = +5 | Treatment: New Treatment Effect = +5 |
| Treatment: New Treatment Effect = +5 | Treatment: New Treatment Effect = +5 | Treatment: New Treatment Effect = +5 | Treatment: New Treatment Effect = +5 |
| Treatment: New Treatment Effect = +5 | Treatment: New Treatment Effect = +5 | Treatment: Current Treatment Effect = -5 | Treatment: Current Treatment Effect = -5 |
| Treatment: Current Treatment Effect = -5 | Treatment: Current Treatment Effect = -5 | Treatment: Current Treatment Effect = -5 | Treatment: Current Treatment Effect = -5 |
| Treatment: Current Treatment Effect = -5 | Treatment: Current Treatment Effect = -5 | Treatment: Current Treatment Effect = -5 | Treatment: Current Treatment Effect = -5 |

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|---------------------------------------|---|---------------------------------------|---------------------------------------|
| Subject 21: Individual Effect = 72 | Subject 22: Individual Effect = 12 | Subject 23: Individual Effect = 22 | Subject 24: Individual Effect = 83 |
| Subject 25: Individual Effect = 93 | Subject 26: Individual Effect = 23 | Subject 27: Individual Effect = 42 | Subject 28: Individual Effect = 62 |
| Subject 29: Individual Effect = 82 | Subject 30: Individual Effect = 102 | Subject 31: Individual Effect = 53 | Subject 32: Individual Effect = 43 |
| Subject 33: Individual Effect = 33 | Subject 34: Individual Effect = 63 | Subject 35: Individual Effect = 73 | Subject 36: Individual Effect = 92 |
| Subject 37: Individual Effect = 13 | Subject 38: Individual Effect = 103 | Subject 39: Individual Effect = 52 | Subject 40: Individual Effect = 32 |