**Chapter 5 Review – Probability**

1. Event A occurs with probability 0.8. The conditional probability that event B occurs, given that A occurs, is 0.5. The probability that both A and B occur is
	1. 0.3. b. 0.4. c. 0.625. d. 0.8. e. need more information
2. If the knowledge that an event A has occurred implies that a second event B cannot occur, the events A and B are said to be
	1. Independent. b. disjoint. c. mutually exhaustive
	d. the sample space. e. complementary
3. You read in a book about bridge that the probability that each of the four players is dealt exactly one ace is about 0.11. This means that
	1. In every 100 bridge deals, each player has one ace exactly 11 times.
	2. In one million bridge deals, the number of deals on which each player has one ace will scarcely be within ±100 of 110,000.
	3. In a very large number of bridge deals, the percent of deals on which each player has one ace will be very close to 11%
	4. In a very large number of bridge deals, the average number of aces in a hand will be very close to 0.11.

Use the following to answer questions 4-5:

A system has two components that operate in parallel, as shown in the diagram below. Because the components operate in parallel, at least one of the components must function properly if the system is to function properly. The probabilities of *failures* for the components 1 and 2 during one period of operation are 0.20 and 0.03 respectively. Let *F* denote the event that the component 1 *fails* during one period of operation and *G* denote the event that component 2 *fails* during one period of operation. The component failures are independent.

4. The event corresponding to the system functioning properly during one period a. F and G b. F or G. c. not F or not G d. not F and not G e. not F or G

 5. The probability that the system functions properly during one period of operation is closest to

 a. 0.5 b. 0.994 c. 0.970 d. 0.940 e. 0.770

 6. In a certain town, 50% of the households own a cellular phone, 40% own a pager, and 20% own both a cellular phone and a pager. The proportion of households that own neither a cell phone nor a pager is

 a. 0% b. 10% c. 30% d. 70% e. 90%

 7. A stack of four cards contains two red cards and two black cards. I select two cards, one at a time, and do *not* replace the first card selected before selecting the second card. Consider the events A = the first card selected is red and B = the second card selected is red; The events A and B are
 a. independent. b. disjoint. c. conditionals. d. complementary. e. none of the above.

 8. Here is an assignment of probabilities to the face that comes up when rolling a die once:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Outcome** | 1 | 2 | 3 | 4 | 5 | 6 |
| **Probability** | 1/7 | 2/7 | 0 | 3/7 | 0 | 1/7 |

Which of the following is true?

1. This isn’t a legitimate assignment of probability, because every face of a die must have a probability of 1/6.
2. This isn’t a legitimate assignment of probability, because it gives probability zero to rolling a
3 or a 5.
3. This isn’t a legitimate assignment of probability, because the probabilities do not add to exactly 1.
4. This isn’t a legitimate assignment of probability, because we must actually roll the die many times to learn the true probabilities.
5. This is a legitimate assignment of probability.

Use the following to answer questions 9-10:

In a particular game, a fair die is tossed. If the number of spots showing is either four or five, you win $1. If the number of spots showing is six, you win $4. If the number of spots showing is one, two, or three, you win nothing. You are going to play the game twice.

 9. The probability that you win at least $1 both times is
 a. 1/2. b. 4/36 c. 1/36. d. 1/4 e. 3/4.

 10. The probability that you win $4 both times is

 a. 1/6 b. 1/3. c. 1/36 d. ¼ e. 1/12

11. When two coins are tossed, the probability of getting two heads is 0.25. This means that
a. of every 100 tosses, exactly 25 will have two heads.
b. the odds against two heads are 4 to 1.
c. in the long run, the average number of heads is 0.25
d. in the long run, two heads will occur on 25% of tosses

12. Event A has probability 0.4. Event B has probability 0.5. If A and B are independent, then the probability that both events occur is
 a. 0.0. b. 0.1. c. 0.2. d. 0.7. e. 0.9.

13. The collection of all possible outcomes of a random phenomenon is called
 a. a census. b. the probability. c. a chance experiment d. the sample space. e. the distribution.

14. The probability of any outcome of a random phenomenon is
 a. the precise degree of randomness present in the phenomenon
 b. the proportion of a very long series of repetitions in which the outcome occurs.
 c. either 0 or 1, depending on whether or not the phenomenon can actually occur.
 d. any number, as long as it is a value between 0 and 1.
 e. impossible to determine if the phenomenon is truly random.

15. The probability that a randomly chosen woman aged 20 to 24 is married is 0.35; the probability that she is widowed or divorced is 0.03. What is the probability that a randomly chosen woman has never been married?
 a. 0.72 b. 0.38 c. 0.62 d. 0.65

**Free Response Questions**

Answer each question completely. All work must be shown to receive full credit.

1. Give an example of two events that are disjointed (mutually exclusive).

Event A and B equal the whole sample space. $P(A and B)^{c} = 0.25$ and $P(A)^{c}=0.35$
2. Draw a Venn diagram for the sample space.

3. Find P(A or B). 4. Find P(A and B).

5. You play tennis regularly with a friend. From past experiences, you believe that the outcome of each match is independent. For any given match, you have a probability of 0.6 of winning. Find the probability that you win the next two matches.

6. Suppose that you have torn a tendon and are facing surgery to repair it. The orthopedic surgeon explains the risks to you. Infection occurs in 3% of such operations, the repair fails in 14% and both infection and failure occur together in 1%. What percent of these operations succeed and are free from infection? Draw a Venn diagram.

7. Parking for students at Central High School is very limited, and those who arrive late have to park illegally and take their chances at getting a ticket. Joey has determined that the probability of having to park illegally and getting a ticket is 0.07. He has kept data from the past year and found that because of his perpetual tardiness, the probability that he will have to park illegally is 0.25. Suppose that he arrived late once again this morning and had to park in the no-parking zone. Find the probability that he gets a ticket.

8. Two cards are dealt, one after the other, for a shuffled 52-card deck. Why is it wrong to say that the probability of getting two red cards is (1/2)(1/2) = ¼? What is the correct probability of this event?

9. Researchers are interested in the relationship between cigarette smoking and lung cancer. Suppose an adult male is randomly selected from a particular population. Assume that the following table shows some probabilities involving the compound event that the individual does or does not smoke and the person is or is not diagnosed with cancer:

|  |  |
| --- | --- |
| **Event** | **Probability** |
| Smokes and gets cancerSmokes and does not get cancerDoes not smoke and gets cancerDoes not smoke and does not get cancer | 0.050.200.030.72 |

Suppose further that the probability that the randomly selected individual is a smoker is 0.25.

1. Find the probability that the individual gets cancer, given that he is a smoker.

1. Find the probability that the individual does not get cancer, given that he is a smoker.
2. Find the probability that the individual gets cancer, given that he does not smoke.
3. Find the probability that the individual does not get cancer, given that he does not smoke.

10. Ivy conducted a taste test for four different brands of chocolate chip cookies. Below is a two-way table that describes which cookie each subject preferred and their gender.

Suppose that one subject from this experiment is selected at random.

1. Find the probability that the selected subject preferred Brand C.
2. Find the probability that the selected person preferred brand C, given that she is a female.

|  |
| --- |
| Cookie Brand |
|   | A | B | C | D | Totals |
| Female | 4 | 6 | 13 | 13 | 36 |
| Male | 22 | 11 | 11 | 14 | 58 |
| Totals | 26 | 17 | 24 | 27 | 94 |

 c. Are the events “preferred Brand C” and “female” independent? Explain.

 d. Are the events “preferred Brand C” and “female” mutually exclusive? Explain.

 e. If a random sample of two subjects is selected, what is the probability that neither preferred Brand C?

11. Approximately 30% of the calls to an airline reservation phone line result in a reservation being made.
 a. Suppose that an operator handles 10 calls. What is the probability that none of the 10 calls results in a reservation?

 b. What assumptions did you make in order to calculate the probability in (a)?

 c. What is the probability that at least one call results in a reservation being made?

12. If three dice are rolled, find the probability of getting triples: i.e., 1, 1, 1 or 2, 2, 2 or 3, 3, 3 etc.

13. Heart disease is the #1 killer today. Suppose that 8% of the patients in a small town are known to have heart disease. And suppose that a test is available that is positive in 96% of patients with heart disease, but is also positive in 7% of patients who do not have heart disease. If a person is selected at random and given the test and it comes out positive, what is the probability that the person actually has heart disease?

14. Officials at Dipstick College are interested in the relationship between participation in (interscholastic) sports and graduation rate. The following table summarizes the probabilities of several events when a male Dipstick student is randomly selected.

|  |  |
| --- | --- |
| **Event** | **Probability** |
| Student participates in sportsStudent participates in sports and graduatesStudent graduates, given no participation in sports | 0.200.180.82 |

1. Draw a tree diagram to summarize the given probabilities above.
2. Find the probability that a student graduates, given that he participates in sports.
3. Find the probability that the individual does not graduate, given that he participates in sports?

1. Find the probability that the student does not participate in sports, given that he graduates.

15. When two dice are rolled, find the probability of getting:

 a. a sum greater than 9

 b. a sum less than 4 or greater than 9

16. A coin is tossed five times.

 a. Find the probability of getting at least one tail.

 b. Find the probability of getting four tails.

17. Suppose you are given a standard six-sided die and told that the die is “loaded” in such a way that while the numbers 1, 3, 4, and 6 are equally likely to turn up, the numbers 2 and 5 are three times as likely to turn up as any of the other numbers. The die is rolled once and the number turning up is observed. Use the information provided to fill in the following table.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Outcome | 1 | 2 | 3 | 4 | 5 | 6 |
| Probability |  |  |  |  |  |  |

a. Let A be the event that the number rolled is a prime number. List the outcomes in A and
find P(A)

 b. Let B be the event that the number rolled is an even number. List the outcomes in B and
 find P(B).

 c. Determine if events A and B are independent.

d. Are events A and B disjoint? Explain briefly.

18. What age groups use social networking sites? A recent study produced the following data about 768 individuals who were asked their age and which of three social networking sites they used most often. (People who did not use such sites were excluded from the study).

|  |  |
| --- | --- |
|  | **Age Group (years)** |
| **Website** | **0 – 24** | **25 – 44**  | **45 – 64**  | **Over 65** | **Totals** |
| **Facebook** | 77 | 105 | 114 | 12 | 308 |
| **Twitter** | 46 | 110 | 81 | 7 | 244 |
| **LinkedIn** | 15 | 97 | 95 | 9 | 216 |
| **Totals** | 138 | 312 | 290 | 28 | 768 |

Suppose one subject from this study was selected at random.

1. Find the probability that the selected subject preferred Twitter.

1. Find the probability that the selected subject preferred Twitter, given that he or she was in the
45 – 64 age group.
2. Are the events “preferred Twitter” and “age group 45 – 64” independent? Explain.
3. Are the events “preferred Twitter” and “age group 45 – 64” mutually exclusive? Explain.

19. Here is the assignment of probabilities that describes the age (in years) and the sex of a randomly selected American college student.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Age** | **14-17** | **18-24** | **25-34** | **>35** |
| **Male** | 0.01 | 0.30 | 0.12 | 0.04 |
| **Female** | 0.01 | 0.30 | 0.13 | 0.09 |

1. What is the probability that the student is a female?

1. What is the conditional probability that the student is a female, given that the student is at least 35 years old?

20. Consider the following experiment: The letters in the word AARDVARK are printed on square pieces of tagboard (same-size squares) with one letter per card. The eight letter cards are then placed in a hat, and one letter card is randomly chosen (without looking) from the hat.

 a. List the sample space S of all possible outcomes.

S = {

 b. Make a table that shows the set of outcomes (X) and the probability of each outcome:

 Outcomes\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
 P(X)

 Consider the following events:

 V: the letter chosen is a vowel.
 F: the letter chosen falls in the first half of the alphabet (i.e., between A and M)

1. List the outcomes in each of the following vents, and determine their probabilities:

V = { P(V) =

F = { P(F) =

V or F = { P(V or F) =

complement of F = { P(Fc) =

1. Determine if events V and F are independent.

21. If four cards are drawn from a standard deck of 52 playing cards and not replaced, find the probability of getting at least one heart.

22. A company is considering implementing one of two quality control plans for monitoring the weights of automobile batteries that it manufactures. If the manufacturing process is working properly, the battery weights are approximately normally distributed with a specified mean and standard deviation. Quality control plan A calls for rejecting a battery as defective if its weight falls more than 2 standard deviations below the specified mean. Quality control plan B calls for rejecting a battery as defective if its weight falls more than 1.5 interquartile ranges below the lower quartile of the specified population. Assume the manufacturing process is under control.

 a. What proportion of batteries will be rejected by plan A?

 b. What is the probability that at least 1 of 2 randomly selected batteries will be rejected by plan A?

 c. What proportion of batteries will be rejected by plan B?