The Airport Problem

At Guadalajara Airport in Mexico, passengers must claim their luggage and then proceed to Customs. In the Customs area, each passenger will press a button that activates a modified stoplight. This light has only red and green bulbs. If the green light shows, the passenger is free to go. If the light turns red, then Customs agents will inspect the passenger's luggage. Customs officials claim that the light has probability 0.30 of showing red on any press of the button.

You are traveling with 6 family members, and you are the seventh person in line.

1. Find the probability that exactly one of the family members in front of you is stopped.

2. Find the probability that more than two of the family members in front of you are stopped.

3. Let X = the number of people in front of you who are stopped. What type of random variable is X? Be very specific, and justify your answer.

4. Find $P(X < 4)$.

5. Find the mean and the variance of X.

6. What is the probability that you are the first person stopped?

Now suppose that there are 50 passengers in the queue for the stoplight. Let's perform a simulation to discover how many of them are stopped by Customs.

7. On your TI-83/84, press MATH, arrow left to PRB, and choose 7:RandBin. Then type 50,0.3) and press ENTER. How many passengers were stopped?

8. The Customs officials claim that the probability that they will stop an individual passenger is, in fact, 0.3 and that individuals are stopped independently of one another. Let's perform a large number of simulations to see what proportion of passengers are stopped if the Customs agents' claim is true. Press MATH, arrow over to PRB, and choose 7:RandBin. Then type 50,0.3) but do not press ENTER yet. We already know that this procedure will give us the number of passengers (out of 50) who will be stopped. What we want, though, is the proportion who will be stopped. To accomplish this, continue typing: /50. This will convert the number of passengers stopped into a proportion. Finally, let's run this simulation 50 times and examine the proportion of passengers stopped each time. Type RandBin(50,0.3,50)/50->L1. It may take a few minutes for the calculator to simulate the trials.

9. Now, let's analyze the proportions that you simulated in Step 8. As always, we begin by making a graphical display. Press 2nd, then Y=, and ENTER. Use the arrow keys to toggle between options in the Statistics Plot menu, and ENTER to select the desired option. Press ZOOM and select 9:ZoomStat to draw the histogram.

10. The calculator chooses an "ideal" window when you use ZoomStat. In reality, this window rarely makes sense. Adjust your window by pressing WINDOW and changing the values to "logical" numbers.

11. Describe your histogram. What does it tell you about the proportion of passengers stopped by Customs for baggage inspection?

12. Next, we should compute numerical summary statistics for our 50 simulations. Press STAT, arrow right to CALC, and choose 1:1-VarStats. Then press 2nd and 1 to type L1. Record your mean and standard deviation. How does the average proportion stopped in your 50 trials compare to the 0.30 claimed by the Customs officials?

13. What if the number of people in the queue changes? Will that affect the distribution of proportion values that you obtain from a simulation? Let's find out. Record your mean and standard deviation each time.
   • First, let there be 100 passengers in the queue for the stoplight. Run a simulation to discover the proportion stopped in each of 50 trials.
   • Repeat the simulation with 150 passengers in line.
   • Now try again with 75 passengers.
   • Finally, use 25 passengers.

14. Share results with your classmates. Complete the table that follows for the standard deviation of the distribution of sample proportions for the given sample size.

Sample size	Standard deviation
25	
50	
75	
100	
150	

15. Make a scatterplot of the standard deviation versus sample size. Describe what you see.

16. Compare your results with the theoretical formula: $\sigma_{\hat{p}} = \sqrt{\dfrac{p(1-p)}{n}}$, where $p = 0.30$.

17. What proportion of a 50-passenger queue would need to be stopped by Customs officials for you to disbelieve their claim of a 0.30 probability of stopping an individual passenger?

Answers: *The Airport Problem*

1. $P(X = 1) = \binom{6}{1}(0.3)^1 (0.7)^5 = 0.302526$ OR   binompdf(6,0.3,1) = 0.3025

2.

$$P(X > 2) = \binom{6}{2}(0.3)^2(0.7)^4 + \binom{6}{3}(0.3)^3(0.7)^3 + \binom{6}{4}(0.3)^4(0.7)^2 + \binom{6}{5}(0.3)^5(0.7)^1 + \binom{6}{6}(0.3)^6(0.7)^0$$

OR

$$P(X > 2) = 1 - P(X \le 2) = 1 - \left(\binom{6}{0}(0.3)^0(0.7)^6 + \binom{6}{1}(0.3)^1(0.7)^5 + \binom{6}{2}(0.3)^2(0.7)^4 \right) = 0.25569$$

OR 1 – binomcdf(6,0.3,2) = 0.2557

3. X is a binomial random variable with parameters $n = 6$ and $p = 0.3$, since the scenario presented meets the 4 criteria for a binomial situation: fixed number of trials, two possible outcomes on each trial: "success" = red and "failure" = green, same probability of success on each trial, and independent trials.

4. $P(X < 4) = P(X \le 3) = \text{binomcdf}(6, 0.3, 3) = 0.92953$

5. $\mu_X = np = 6(0.3) = 1.8$ \qquad $\sigma_X = \sqrt{np(1 - p)} = \sqrt{6(0.3)(0.7)} = 1.1225$

6. This is a geometric situation, since we are waiting on the first "success." For this geometric distribution, $p = 0.3$. Let Y = number of trials up to and including the first person stopped.
   $P(Y = 7) = (0.7)^6(0.3) = 0.0353$ OR   geometpdf(0.3,7)=0.0353

7. In our simulation, randBin(50,0.3) = 22.

8.

The Practice of Statistics, 4/e- Chapter 7 © 2011 BFW Publishers