6.3 - Binomial & Geometric Random Variables

Notes provided by E. Kelly Pendleton from Ardrey Kell High School, including:

-binomial setting & binomial random variables
-binomial probabilities
-mean and standard deviation of a binomial distribution
-geometric random variables

adjust as you wish--this is typically broken up over 2 days + one extra day of practice
email elizabethk.pendleton@cms.k12.nc.us if you need the flipchart version

Warm-Up (2001 #2)

2. A department supervisor is considering purchasing one of two comparable photocopy machines, *A* or *B*. Machine *A* costs \$10,000 and machine *B* costs \$10,500. This department replaces photocopy machines every three years. The repair contract for machine *A* costs \$50 per month and covers an unlimited number of repairs. The repair contract for machine *B* costs \$200 per repair. Based on past performance, the distribution of the number of repairs needed over any one-year period for machine *B* is shown below.

Number of Repairs	0	1	2	3
Probability	0.50	0.25	0.15	0.10

You are asked to give a recommendation based on overall cost as to which machine, *A* or *B*, along with its repair contract, should be purchased. What would your recommendation be? Give a statistical justification to support your recommendation.

6.3 Binomial & Geometric Distributions

Part 1: Binomial Distribution

The binomial random variable X is defined as the number of successes out of a fixed number of trials

1. **B**inary - Each trial results in one of two mutually exclusive outcomes. (success/failure)
2. **I**ndependent - Outcomes of different trials are independent.
3. **N**umber of trials is fixed.
4. P(**S**uccess) is the same for each trial.

Determine if the following scenarios are binomial. If they are, identify the number of trials and the probability of success for each trial. Let n = # of trials & p = probability of success.

1) Harper Hendricks has a 75% free throw percentage. Count the number of free throws made out of 10 tries.

2) Jeremy Littlejohn has a 60% field goal percentage (shots made from different spots on the court). Count the number of shots made out of 10 tries.

3) Deal 10 cards from a shuffled deck and count the number of red cards.

4) Roll a die and count the number of rolls until the first three.

Reminder from Chapter 5:
combinations - the number of ways to get k successess out of n trials

$$_nC_k$$

How many ways can you make 6 free throws out of 10 attempts?

With a free throw percentage of 75%, find the probability that Harper makes 6 of 10 free throws.

Binomial Formula

$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$$

k = successes
n = trials
p = probability of success

30% of students find homework answers online and copy them. Suppose I check the homework of 5 students. Find the probability that:
a). exactly one student copied from the Internet.

b). at most 1 student copied from the Internet.

Using the Calculator

binompdf = a single probability [P(X = k)]

- n = # of trials
- p = probability of success
- X = k

binomcdf = cumulative probability [P(X ≤ k)]

- n = # of trials
- p = probability of success
- X ≤ k

*your calculator can only do = or ≤
*any other inequality needs to be adjusted accordingly

Suppose that a certain cereal brand advertizes that 15% of its boxes have a coupon for a free FitBit. You buy 6 boxes of this cereal to see if you can get a free FitBit.

a). Describe the variable X.

What is the probability that you will get:
b). exactly 2 FitBits?

c). at most 2 FitBits?

d). at least 2 FitBits?

e). less than 4 FitBits?

f). more than 4 FitBits?

g). How many FitBits do you expect to win when buying 6 boxes?

h). Give the probability distribution of X.

X	
P(X)	

Describe the distribution. [Reminder: Center, Shape, Spread]

Mean & Standard Deviation of a Binomial Random Variable

$$E(x) = \mu_x = np$$

$$\sigma_x = \sqrt{np(1-p)}$$

Discrete Distributions Review Worksheet [supplemental materials provided in Activities in the Canvas course]

-This is the back of your warm-up. Complete all problems. Show all work as if it were an FRQ.

HW: pg. 403 #72-75, 77, 80-83, 86

Binomial Worksheet #2

[supplemental materials provided in Activities in the Canvas course]

Take out pg. 403.

6.3 Part 1 Continued

Normal Approximation for Binomial Distributions

If $np \geq 10$ and $n(1 - p) \geq 10$, then n is large and the binomial distribution of X is approximately Normal:

$$N\left(\text{np}, \sqrt{\text{np}(1-p)}\right)$$

When an opinion poll calls residential telephone numbers at random, only 20% of the calls reach a person. The company makes 1000 calls using a random digit dialing machine.

1. Find the mean and standard deviation of the distribution.

2. Is a Normal approximation appropriate for this situation? Why or why not?

3. Find the probability that the calls reach between 175 and 215 people.

6.3 Part 2: Geometric Distribution

The geometric random variable X is defined as the number of trials required to obtain the first success

1. Binary - Each trial results in one of two mutually exclusive outcomes. (success/failure)
2. Independent - Outcomes of different trials are independent.
3. Trials are counted until the First success.
4. P(Success) is the same for each trial.

What's the difference between binomial and geometric? What should you look for in a problem?

10% of students forget to bring their ID to school. Mr. Switzer randomly checks for IDs when he enters a classroom and counts students until he finds the first person without an ID.

P(X = 1)
P(X = 2)
P(X = 3)
P(X = 4)
P(X = 5)
P(X = n)

X	
P(X)	

How many should he expect to meet before he encounters his first student without an ID?

Geometric Formulas

$$P(X = n) = (1 - p)^{n-1} \cdot p$$

$$\mu_X = \frac{1}{p}$$

$$P(X > n) = (1-p)^n$$

Facts About Geometric Distributions

****All geometric distributions are skewed _____.****
-because you multiply by (1-p), a number less than 1, to get to the next term, the probabilities only decrease.

A geometric distrubtion is the same as a geometric sequence (from Precalculus).

Even though it is infinite, the probabilities sum to 1 because:

$$\frac{a_1}{1-r} = 1$$

7% of US adults believe chocolate milk comes from brown cows. (Source: Washington Post)
a. What is a trial in this situation?

What is the probability that:
b. the first person that believes this is the third person to be interviewed?

c. we find someone who believes this by the third shot person?

d. it will take more than 3 interviews to find someone who believes this.

e. it will take at least 5 people to find someone who believes this.

f. What is the mean?

g. Give the distribution of the first 5 values and draw a histogram.

Classwork: **Finish the worksheet, starting with "Geometric Distribution WS" and then finish the "Discrete Distributions Worksheet" (on the back of the warm-up from last class).**

[supplemental materials provided in Activities in the Canvas course]

HW: **pg. 405 #96-105**