6.1 - Discrete & Continuous Random Variables

Notes provided by E. Kelly Pendleton from Ardrey Kell High School, including:

-discrete random variables
-mean and standard deviation of a discrete random variable
-continuous random variables

adjust as you wish
email elizabethk.pendleton@cms.k12.nc.us if you need the flipchart version

Warm-Up

A pharmaceutical company manufactures capsules that contain an average of 507 grams of vitamin C. The standard deviation is 3 grams. The distribution of vitamin C is considered to be normal amongst the capsules.

1. What percent of the capsules will have above 512 grams?

Reminder: use 2nd-VARS-2: Normalcdf

$P(X>512)$

$\sigma=3$ $=0.0478$

2. What percent of the capsules will have less than 500 grams?

$P(X<500)=0.98\%$

3. What percent will have more than 600?

$P(X>600)=0$

CHAPTER 6 - Random Variables

random variable - a variable whose value is a chance numerical outcome of a random phenomenon

*use capital letters at the end of the alphabet to represent random variables.

EX) Toss a coin two times and count the number of tails: Let random variable X = # of tails

$X = \{0, 1, 2\}$

Let random variable Y = any number between 0 and 1.

$0 \leq Y \leq 1$

continuous random variable - a random variable that takes on all values in an interval of real numbers (there are an infinite number of outcomes).

- generate a random number between 0 and 1
- time
- normal distributions

can be MEASURED

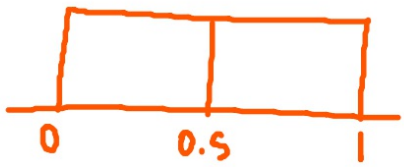
Probability Model for X
assigns the probability P(A) equal to the area above A and under a curve where

1. $p(x) \geq 0$ for all x.
2. the total area under the graph of $p(x)$ is 1

can be described by a density curve

* All continuous probability distributions assign probability 0 to individual outcomes.

Ex). Generate a random number between 0 and 1. Find each probability:



a. $P(X = 0.5) = 0$

$\frac{1}{\infty} = 0$

b. $P(0.3 \leq X \leq 0.7)$

$= 0.4$

c. $P(X \leq 0.8)$

$= 0.8$

d. $P(X < 0.8)$

0.8

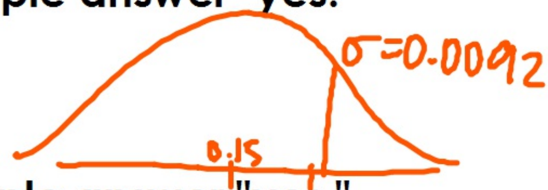
e. $P(X < 0.5 \text{ or } X > 0.8)$

$0.5 + 0.2 = 0.7$

An SRS of 1500 adults were asked, "Do you happen to jog?" Suppose that in fact 15% of all adults would answer "yes." The distribution of responses is approximately normally distributed with a mean of 0.15 and a standard deviation of 0.0092. Find the probabilities of the following events: $N(0.15, 0.0092)$

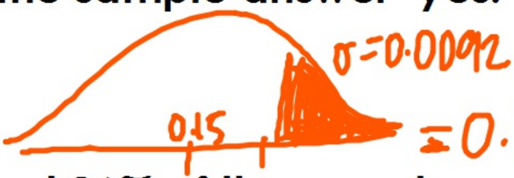
a. exactly 16% of the sample answer "yes."

$P(X = 0.16) = 0$



b. at least 16% of the sample answer "yes."

$P(X \geq 0.16)$



$= 0.1385$

c. between 14% and 16% of the sample answer "yes."

$P(0.14 < X < 0.16) = 0.7229$

discrete random variable - a random variable that takes on a finite number of values (you can count the number of outcomes).

probability distribution: lists the values of X and their probabilities where

1. $0 \le p_i \le 1$
2. $p_1 + p_2 + p_3 + \ldots + p_k = 1$

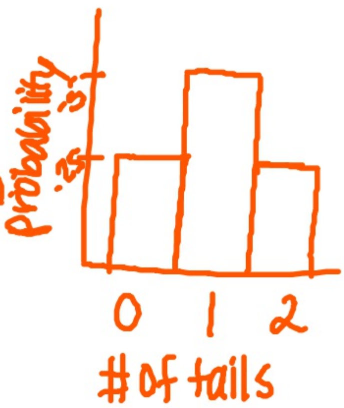
Ex). Toss two coins. Create a probability model for the number of tails. $X = \#$ of tails

HH HT TH TT

table:

X	0	1	2
$P(X)$	1/4	2/4	1/4

histogram:



A couple plans to have three children. There are 8 possible arrangements of boys and girls. For example, GGB means the first two children are girls and the third child is a boy. All 8 arrangements are (approximately) equally likely. $2 \cdot 2 \cdot 2 = 8$

a. List the possible arrangements of boys and girls if the couple has 3 children.

$\overset{3}{GGG}$, $\overset{0}{BBB}$, $\overset{2}{GGB}$, $\overset{2}{BGG}$, $\overset{1}{BGB}$, $\overset{2}{GBG}$, $\overset{1}{GBB}$, $\overset{1}{BBG}$

b. Let X be the number of girls the couple has. Create the probability distribution for X. X = # of girls

X	0	1	2	3
P(X)	1/8	3/8	3/8	1/8

Find the probability that:

c. the couple will have exactly two girls.

$P(X=2) = 3/8$

d. the couple will have at least 2 girls.

$P(X \geq 2) = 3/8 + 1/8 = 4/8$

e. the couple will have less than 2 girls.

$P(X < 2) = 1/8 + 3/8 = 4/8$

Center & Spread of a Discrete Random Variable

Center

Expected value - mean of a random variable X

$E(X) = \mu_x = \Sigma x_i p_i$ → formula sheet

$= x_1p_1 + x_2p_2 + \cdots + x_kp_k$

X	0	1	2	3
P(X)	0.125	0.375	0.375	0.125

$E(X) = \mu_x = 0(.125) + 1(.375) + 2(.375) + 3(.125)$

$= 1.5$ girls   DON'T ROUND!

In the long run, we expect a family w/ 3 children to have an average of 1.5 girls

Spread

Variance of a random variable: average squared deviation

$$\text{Var}(X) \;\; or \;\; \sigma^2 = \Sigma(x_i - \mu_x)^2 p_i$$ →formula sheet

$$= (x_1 - \mu_x)^2 p_1 + (x_2 - \mu_x)^2 p_2 + \cdots + (x_k - \mu_x)^2 p_k$$

Standard deviation: $\sigma_x = \sqrt{\sigma_x^2}$ → st. dev. = $\sqrt{\text{variance}}$

X	0	1	2	3
P(X)	0.125	0.375	0.375	0.125

$$\text{Var}(x) = \sigma_x^2 = (\overset{x}{0} - \overset{\mu}{1.5})^2(\overset{p}{.125}) + (1-1.5)^2(.375) + (2-1.5)^2(.375) + (3-1.5)^2(.125) = 0.75 \text{ girls}^2$$

$$\sigma_x = \sqrt{0.75} = 0.87 \text{ girls}$$

Each family deviates by about 0.87 girls from the average of 1.5 girls

Calculator

L1: x-values

L2: probabilities

Stat → Calc → 1-var stats

List: L1

FreqList: L2

how to show work:

$$\mu_x = x_1p_1 + \ldots + x_kp_k$$

$$0(.125) + \ldots + 3(.125) = 1.5 \text{ girls}$$

$$\sigma_x = \sqrt{\Sigma(x_i - \mu)^2 p_i}$$

$$= \sqrt{(0-1.5)^2(.125) + \ldots + (3-1.5)^2(.125)} = 0.87$$

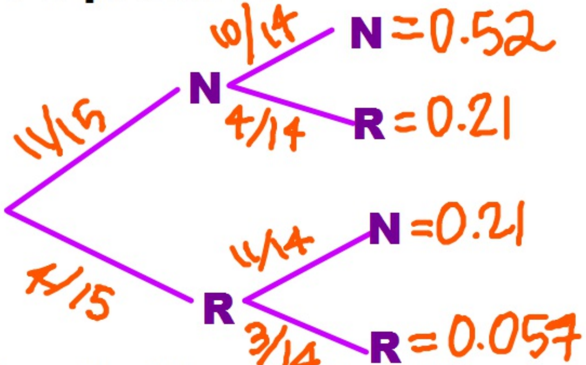
Using the Calculator

As the head of inventory for Knowway computer company, you were thrilled that you had managed to ship 2 computers to your biggest client the day the order arrived. You are horrified, though, to find out that someone restocked refurbished computers in with the new computers in your storeroom. The shipped computers were selected randomly from the 15 computers in stock, but 4 of those were actually refurbished. If your client gets 2 new computers, things are fine. If the client gets a refurbished computer, it will be sent back at your expense ($100) and you can replace it. However, if both computers are refurbished, the client will cancel the order this month and you'll lose $1000. What's the expected value and standard deviation of your loss?

Define the random variable: X = amount of loss

Make a picture:

R = 4/15
N = 11/15



Create the probability model:

X	0	100	1000
P(X)	0.52	0.42	0.06

Find the expected value: $E(X) = 0(.52) + \ldots + 1000(.06) = \$102$

Find the standard deviation:

$$\sigma_X = \sqrt{(0-102)^2(.52) + \ldots + (1000-102)^2(.06)}$$

$231.94

State your conclusion:

I expect this mistake to cost the firm about $102 with a standard deviation of $231.95.